



(handout slides uploaded to conference app/website)

“Helping English Language Learners Navigate
Probability Vocabulary and Concepts”
Amy Wagler & Larry Lesser
The University of Texas at El Paso
75-minute breakout based largely on our paper (with Berenice Salazar)
in the November 2016 *Statistics Education Research Journal*



Definition of ELL

students who experience “enough limitations that he or she cannot fully participate in mainstream English instruction” (Goldenberg, 2008, p. 10), which includes those beginning to learn English who could benefit from language support and those who are proficient in English but may need additional assistance in social or academic situations (Hoffstetter, 2003).

our work on language/ELLs in statistics

<http://www.math.utep.edu/Faculty/lesser/ELL.html>

- 2009 *SERJ*: case study of two ELLs
- 2013 *SERJ*: CLASS survey
- 2015 *J. of Technical Writing and Communication*: readability of a corpus of college statistics textbooks
- 2016 *J. of Computers in Mathematics and Science Teaching*: tools to assess readability of teaching materials
- Nov. 2016 *SERJ*: case study of ELLs using bilingual probability applet

importance/rationale for topic

- Importance of language
- Spanish is the second-most spoken language in the world and is by far the most common language of ELLs in US
- ELL-friendly teaching practices can help *all* students

Question for group discussion

What are some examples of how **language** can be challenging for any student learning probability?

Language in probability:
negation location

Pr(all rolls are **not** 5's)

versus

Pr(**not** all rolls are 5's)

Language in probability:
conditional probability

Probability of someone testing positive having cancer

versus

Probability of someone having cancer testing positive

Language in probability:
specifying sets of discrete events

Sullivan (2010):

Section 6.2 The Binomial Probability Distribution 347

Binomial Probability Distribution Function
The probability of obtaining x successes in n independent trials of a binomial experiment is given by

$$P(x) = {}_n C_x p^x (1-p)^{n-x} \quad x = 0, 1, 2, \dots, n \quad (1)$$

where p is the probability of success.

While reading probability problems, pay special attention to key phrases that translate into mathematical symbols. Table 9 lists various phrases and their corresponding mathematical equivalent.

Phrase	Math Symbol
"at least" or "no less than" or "greater than or equal to"	\geq
"more than" or "greater than"	$>$
"fewer than" or "less than"	$<$
"no more than" or "at most" or "less than or equal to"	\leq
"exactly" or "equals" or "is"	$=$

Language in probability: lexical ambiguity (e.g., the word **random**)

Kaplan, Rogness, Fisher (2014)




Figure 1a. Random Zebras (Colloquial).




Figure 1b. Random Hat (Statistical).

Question for group discussion

What are some examples of how **culture** can play a role in how a student learns probability?

Culture in probability

- differences on nature or role of **randomness** (Eglash, 2005)
- Culturally-relevant **games** (e.g., Toma Todo, la lotería, etc. vs. card games, etc.)
- Manipulatives**: "fair die", "draw a card", sides of a coin, "faces" of a coin


Culture in probability: Manipulatives

- "fair die"**: not knowing what a 'fair die' was, an K-12 ELL could not answer "If you rolled a fair die, what is the probability of getting a number less than 3?" (Yu Ren Dong, March 2016 Mathematics Teacher)
- "draw a card"**: two 3rd-grade students drew the 6 of spades in their math notebook (http://article.illustrativemathematics.org/0214/11/teacher-differences.html)
- sides of a coin**: college ELL interview excerpt (from Lesser & Winsor, 2009):
M: The second event is 'quarter lands on tails.'
S2: What is *tails* on the quarter?
[Mexican coins: seal (or sun) and eagle;
other Latin America: cara[face] y cruz[cross], shield, crown]

tossing (asymmetric) moon blocks

(can disrupt equiprobability bias)

- each crescent-shaped block has flat(*yang*) and curved(*yin*) sides
- used in China, Hong Kong, Taiwan, etc. to indicate - (2 yins) or + (1 of each) fortune
- What's the probability of the latter?



Specific Context: Coin Flipping

- Real-world:** decided some precinct delegates in Iowa political caucuses, opening NFL kickoffs, etc.
- Statistics:** flips are Bernoulli trials, have the simplest equiprobable sample space, are a benchmark for randomness, etc.
- Probability education research** reviewed in our 2016 *SERJ* paper: Falk & Lann (2015), Rubel (2007), Sedlmeier (1998), Watson & English (2015), etc.

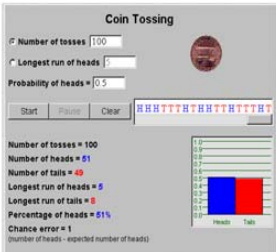
Coin-flipping can illustrate probability misconceptions

misconception	Example: A person believes....
Equiprobability bias	"exactly 3 heads" or "exactly 1 head" are equally likely for a 3-flip sequence
Gambler's fallacy	after 9 heads, the 10 th toss is more likely to be tails
Law of Small Numbers	even short runs of coin flips to reflect the fairness of a coin
Representativeness Heuristic	a sequence of coin tossing with a very long streak of heads or with a well-ordered pattern such as THHTHTH is not representative of a random process
Availability Heuristic	there are more 10-flip sequences with exactly 2 heads than with exactly 8 heads

multilingual probability resources

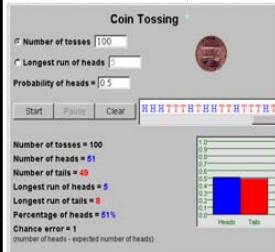
- Terms in 29+ languages at <http://isi.cbs.nl/glossary>
- Multilingual collections of **applets** (e.g., NLVM or Shodor)

Google the words **NLVM coin tossing applet**



Number of tosses = 100
 Number of heads = 61
 Number of tails = 39
 Longest run of heads = 5
 Longest run of tails = 8
 Percentage of heads = 61%
 Chance error = 1

explore the **NLVM coin tossing applet** in English, Spanish, French, or Chinese!



Cant. de lanzamientos = 100
 Número de caras = 48
 Número de escudos = 52
 Mayor número de caras sucesivas = 4
 Mayor número de escudos sucesivos = 6
 Porcentaje de caras = 48%
 Error aleatorio = 2

questions adapted from protocol

- What is the **longest run** in this sequence?
H T T H H T H H H T T H
- For a 100-flip sequence, how long do you think the **longest run** will be?

Research Questions

- What is the nature of how Spanish-speaking ELLs use a bilingual applet when learning probability?
- When does it appear that language plays a factor when Spanish-speaking ELLs explore probability with the applet?

timeline

STEP	DATES
Study design, IRB process	Oct. 2011 – Feb. 2012
Recruitment from intro. stat. students	Feb. – March 2012
Interviews (n = 6 ♀)	March – April 2012
Interview transcription	June – July 2012
Analysis	August – December 2012
Peer debriefing (by 19 mathematics education grad. student researchers)	Oct. 15, 2012; April 22, 2015
Final refinements	April – May 2015
article published in SERJ	Nov. 2016

Mock Interviews

- In the next slide, a set of three questions from the protocol are provided
- Do the following:
 - Choose roles (interviewer, ELL interviewee, non-ELL interviewee, recorder)
 - Stay in character during interviews
 - Discuss results (out of character)
 - Debrief

Mock Interviews

En tus propias palabras, ¿Que significa la "mayor racha" o el mayor número de caras sucesivas?

En tus propias palabras, ¿Que significa "a largo plazo"?

En la secuencia A, ¿Cuál es la mayor racha o el mayor número de caras sucesivas?

En la secuencia B, ¿Cuál es la mayor racha o el mayor número de caras sucesivas?

Para una secuencia de 100-lanzamientos, ¿Qué tan larga crees que sea la racha más larga de número de caras o escudos?

Secuencia A:
C E E E C E E C E C E E E C E E E C E E E C E E C E E C E E E C E E E C E
Secuencia B:
C E C E E C C E C E C C E E C E E C C E E C C E E C E C E C E C E C E C E

Debrief

- Points of consensus:
- Questions that arose:
- Themes:

from Lesser, Wagler, & Salazar (2016)

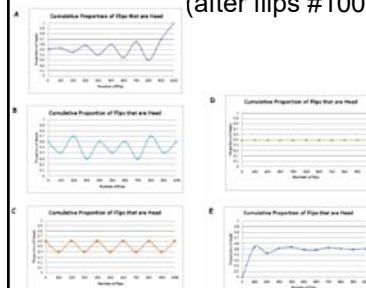
B: in your own words...what does 'longest run' mean to you?

P1: ...the more, the most, hmmm, the fastest to flip the coin, like [short pause] many times but so fast [nervous laugh]

question adapted from protocol

Sketch a plausible graph of the **cumulative proportion of flips that are heads**

(after flips #100, 200, 300, ..., etc.)



Milo Schield shared that a record 645+ meanings of "run" were found by Oxford English Dictionary lexicographer Peter Gilliver

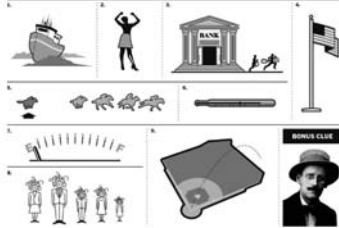
A Verb for Our Frantic Times

www.nytimes.com

By SIMON WINCHESTER

THIS MORNING'S VOCABULARY QUIZ

INSTRUCTIONS: Each of the numbered items below refers to one of the hundreds of words in the O.E.D.'s definition of "run."



ANSWERS: 1. Run aground. 2. A run in her stocking. 3. Take the money and run. 4. Run it up the flagpole. 5. Adulter. 6. Run a fever. 7. Running on empty. 8. Runs in the family. 9. Home run. BONUS: "Along the riverbank" from "Purgatory House" by James Joyce. Sam Poole.

Multiple meanings of **run** in statistics

- Difference of x-coordinates (e.g., slope is "rise over **run**")
- A sequence of at least 2 consecutive identical outcomes (e.g., "what is the longest **run** of heads?")
- In the long **run**
- **Run** the [experiment/simulation/program]

Multiple meanings of **run** in statistics

What is a run in a designed experiment?

Learn more about MinTab 17

A run is an experimental condition or factor level combination at which responses are measured. Usually, each run corresponds to a row in the worksheet and results in one or more response measurements, or observations. For example, you do a full factorial design with two factors, each with two levels. Your experiment has four runs.

Run	Factor 1	Factor 2	Response
1	-1	-1	11
2	1	-1	12
3	-1	1	10
4	1	1	9

NOTE

When doing an experiment, the run order should be randomized.

Each run corresponds to a design point, and the entire set of runs is the design. Multiple executions of the same experimental conditions are considered separate runs and are called replicates.

Language recommendations (using Spanish as a resource!)

- replace "in the long run" by "in the long term" (*en el largo plaza*); Sullivan (2010) uses "long-term proportion"
 - replace "longest run of heads" by "largest number of successive [consecutive] heads" (*el mayor número de caras sucesivas*)
- Also,
- replace "face of the coin" by "side of the coin" (to avoid confusion with *cara*[heads])

Another reason to distinguish similar-sounding phrases

- “long run” and “longest run”
- Mean, median, mode: Lesser & Winsor (2009) & CLASS survey

Pedagogical discussion

- Visuals (e.g., the sequence of flips, the bar chart of flips accumulated) help!

nlvm.usu.edu/es/nav/vlibrary.html

eduteka.org/MI/master/interactiva/

<http://isi.cbs.nl/glossary/>

Language	Description
English	arithmetic mean, arithmetic average, mean, average
French	moyenne arithmétique, moyenne, valeur moyenne, valeur médio
German	arithmetisches Mittel, arithmetisches Mittelwert, Durchschnitt, Mittelwert, Mittel
Dutch	rekundkundige gemiddelde, gemiddelde
Italian	media aritmetica, media, valore medio
Spanish	media aritmética, promedio aritmético, promedio, media
Catalan	migjana aritmètica, valor mitjà, mitjana
Portuguese	média aritmética, média
Romanian	media aritmetică
Danish	aritmetisk gennemsnit, middelværdi, gennemsnit, middelv
Norwegian	aritmetisk gjennomsnitt, forsvinnings, aritmetisk middel, gjennomsnitt
Swedish	aritmetiskt medelvärde
Czech	aritmetický průměr
Finnish	aritmetiittien keskiarvo, keskiarvo, keskiarvo, keskiarvo (japannila)
Hungarian	számtani átlag, középérték
Turkish	aritmetik ortalama, ortalama
Estonian	aritmeetiline keskmine, keskmine, keskmine
Lithuanian	aritmetinis vidurkis, vidurinė reikšmė, vidurkis
Slovenian	aritmetična sredina, enostavna aritmetična sredina, srednja vrednost, povprečna, povprečna vrednost, sredina
Polish	średnia arytmetyczna, średnia, przeciętna, obliczar, przeciętna, obliczar średnią, sławiec średnią, średnio, przeciętna (średni, jako zero)
Russian	Среднее арифметическое
Ukrainian	середнє арифметичне
Serbian	aritmetična sredina, aritmetični prosek, sredina, prosek
Croatian	međunarodni: bareni međunarodni, eritmetički, samostojni međunarodni, bareni međunarodni, rekognoski međunarodni, međunarodni, vanostajni, glib, vanostajni, vanostajni
Euskara	aritmetikoko erdiaritmetiko, baita, baita, baita
Farsi	میانگین حسابی, میانه, میانگین
Parisi-Farsi	میانگین حسابی
Arabic	المتوسط الحسابي, المتوسط الحسابي
Afrikaans	rekundkundige gemiddelde, gemiddelde
Chinese	算术平均 (值), 算数平均
Korean	산술 평균

What are your questions ...or suggestions?

- Our ELL work and URLs of the resources: www.math.utep.edu/Faculty/lesser/ELL.html

- Contact us: Lesser@utep.edu or awagler2@utep.edu



