(handout slides uploaded to conference app/website)

"Helping English Language Learners Navigate Probability Vocabulary and Concepts"

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The University of Texas at El Paso
75-minute breakout based largely on our paper (with Berenice Salazar)
in the November 2016 *Statistics Education Research Journal*

UTEP

Definition of ELL

students who experience "enough limitations that he or she cannot fully participate in mainstream English instruction" (Goldenberg, 2008, p. 10), which includes those beginning to learn English who could benefit from language support and those who are proficient in English but may need additional assistance in social or academic situations (Hoffstetter, 2003).

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our work on language/ELLs in statistics
http://www.math.utep.edu/Faculty/lesser/ELL.html

- 2009 *SERJ*: case study of two ELLs
- 2013 *SERJ*: CLASS survey
- 2015 *J. of Technical Writing and Communication*: readability of a corpus of college statistics textbooks
- 2016 *J. of Computers in Mathematics and Science Teaching*: tools to assess readability of teaching materials
- Nov. 2016 *SERJ*: case study of ELLs using bilingual probability applet

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importance/rationale for topic

- Importance of language
- Spanish is the second-most spoken language in the world and is by far the most common language of ELLs in US
- ELL-friendly teaching practices can help *all* students

Question for group discussion

What are some examples
of how language can be challenging
for any student learning probability?

Language in probability:
negation location

Pr(all rolls are not 5's)

versus

Pr(not all rolls are 5's)

Language in probability: conditional probability

Probability of someone testing positive having cancer

versus

Probability of someone having cancer testing positive

Language in probability: specifying sets of discrete events

Sullivan (2010):

Language in probability: lexical ambiguity (e.g., the word *random*)

Kaplan, Rogness, Fisher (2014)

Figure 1a. Random Zebras (Colloquial).

Figure 1b. Random Hat (Statistical).

Question for group discussion

What are some examples
of how culture can play a role
in how a student learns probability?

Culture in probability

- differences on nature or role of randomness (Eglash, 2005)
- Culturally-relevant games (e.g., Toma Todo, la lotería, etc. vs. card games, etc.)
- Manipulatives: "fair die", "draw a card", sides of a coin, "faces" of a coin

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Culture in probability: Manipulatives

- "fair die": not knowing what a 'fair die' was, an K-12 ELL could not answer "If you rolled a fair die, what is the probability of getting a number less than 3?" (Yu Ren Dong, March 2016 *Mathematics Teacher*)
- "draw a card": two 3rd-grade students drew the 6 of spades in their math notebook
  http://ericarthurmiller.blogspot.com/2014/11/learner-differences.html
- sides of a coin:

college ELL interview excerpt (from Lesser & Winsor, 2009):

M: The second event is 'quarter lands on tails.'

S2: What is *tails* on the quarter?

[Mexican coins: seal (or sun) and eagle;
 other Latin America: cara[face] y cruz[cross], shield, crown]

tossing (asymmetric) moon blocks
(can disrupt equiprobability bias)

- each crescent-shaped block has flat(*yang*) and curved(*yin*) sides
- used in China, Hong Kong, Taiwan, etc. to indicate - (2 yins) or + (1 of each) fortune
- What's probability of the latter?

Specific Context: Coin Flipping

- **Real-world**: decided some precinct delegates in Iowa political caucuses, opening NFL kickoffs, etc.
- **Statistics**: flips are Bernoulli trials, have the simplest equiprobable sample space, are a benchmark for randomness, etc.
- **Probability education research** reviewed in our 2016 *SERJ* paper: Falk & Lann (2015), Rubel (2007), Sedlmeier (1998), Watson & English (2015), etc.
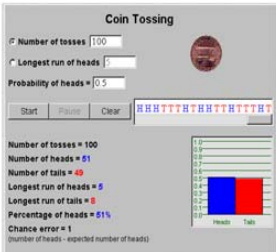
Coin-flipping can illustrate probability misconceptions

misconception	Example: A person believes….
Equiprobability bias	"exactly 3 heads" or "exactly 1 head" are equally likely for a 3-flip sequence
Gambler's fallacy	after 9 heads, the 10th toss is more likely to be tails
Law of Small Numbers	even short runs of coin flips to reflect the fairness of a coin
Representativeness Heuristic	a sequence of coin tossing with a very long streak of heads or with a well-ordered pattern such as THTHTHTH is not representative of a random process
Availability Heuristic	there are more 10-flip sequences with exactly 2 heads than with exactly 8 heads

multilingual probability resources

- Terms in 29+ languages at http://isi.cbs.nl/glossary

- Multilingual collections of **applets** (e.g., NLVM or Shodor)

Google the words **NLVM coin tossing applet**

explore the **NLVM coin tossing applet** in English, Spanish, French, or Chinese!

questions adapted from protocol

- What is the longest run in this sequence?
H T T H H T H H H T T H
- For a 100-flip sequence, how long do you think the longest run will be?

Research Questions

- What is the nature of how Spanish-speaking ELLs use a bilingual applet when learning probability?

- When does it appear that language plays a factor when Spanish-speaking ELLs explore probability with the applet?

timeline

STEP	DATES
Study design, IRB process	Oct. 2011 – Feb. 2012
Recruitment from intro. stat. students	Feb. – March 2012
Interviews ($n = 6$ ♀)	March – April 2012
Interview transcription	June – July 2012
Analysis	August – December 2012
Peer debriefing (by 19 mathematics education grad. student researchers)	Oct. 15, 2012; April 22, 2015
Final refinements	April – May 2015
article published in SERJ	Nov. 2016

Mock Interviews

- In the next slide, a set of three questions from the protocol are provided
- Do the following:
  - Choose roles (interviewer, ELL interviewee, non-ELL interviewee, recorder)
  - Stay in character during interviews
  - Discuss results (out of character)
  - Debrief

Mock Interviews

En tus propias palabras, ¿Que significa la "mayor racha" o el mayor número de caras sucesivas?

En tus propias palabras, ¿Que significa "a largo plazo"?

En la secuencia A, ¿Cuál es la mayor racha o el mayor número de caras sucesivas?
En la secuencia B, ¿Cuál es la mayor racha o el mayor número de caras sucesivas?
Para una secuencia de 100-lanzamientos, ¿Qué tan larga crees que sea la racha más larga de número de caras o escudos?

Sequencia A:
C E E C E C E C E E C E E C C E E E C E E C E E C E E E C E E E C E
Sequencia B:
C E C E E C C E C E C C E E C E E C C E E C E C C E E C E C E C E C E C E C E

Debrief

- Points of consensus:

- Questions that arose:
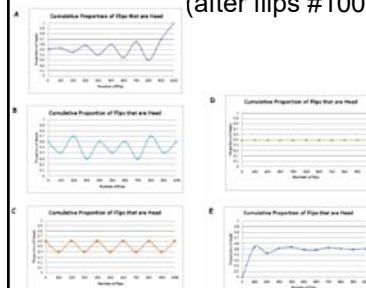
- Themes:

from Lesser, Wagler, & Salazar (2016)

B: in your own words…what does 'longest run' mean to you?

P1: …the more, the most, hmmmm, the fastest to flip the coin, like [short pause] many times but so fast [nervous laugh]

question adapted from protocol

Sketch a plausible graph of the cumulative proportion of flips that are heads

(after flips #100, 200, 300,….,etc.)

Milo Schield shared that a record 645+ meanings of "run" were found by Oxford English Dictionary lexicographer Peter Gilliver
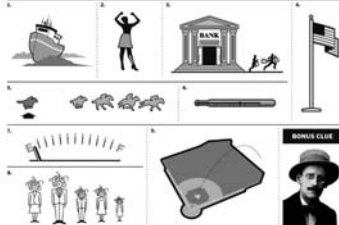
A Verb for Our Frantic Times

www.nytimes.com

By SIMON WINCHESTER

THIS MORNING'S VOCABULARY QUIZ

INSTRUCTIONS: Each of the numbered clues below refers to one of the hundreds of senses in the O.E.D.'s definition of "run."

BONUS CLUE

ANSWERS: 1. Run aground. 2. A run in her stocking. 3. Take the money and run. 4. Run it up the flagpole. 5. Also-ran. 6. Run a fever. 7. Running on empty. 8. Runs in the family. 9. Home run. BONUS: "Along the riverrun" from "Finnegans Wake" by James Joyce. Sam Potts.

Multiple meanings of run in statistics

- Difference of *x*-coordinates
  (e.g., slope is "rise over run")
- A sequence of at least 2 consecutive identical outcomes (e.g., "what is the longest run of heads?")
- In the long run
- Run the [experiment/simulation/program]

Multiple meanings of run in statistics

What is a run in a designed experiment?

Learn more about Minitab 17

A run is an experimental condition or factor level combination at which responses are measured. Usually, each run corresponds to a row in the worksheet and results in one or more response measurements, or observations. For example, you do a full factorial design with two factors, each with two levels. Your experiment has four runs:

Run	Factor 1	Factor 2	Response
1	-1	-1	11
2	1	-1	12
3	-1	1	10
4	1	1	9

NOTE
When doing an experiment, the run order should be randomized.

Each run corresponds to a design point, and the entire set of runs is the design. Multiple executions of the same experimental conditions are considered separate runs and are called replicates.

Language recommendations (using Spanish as a resource!)

- replace "in the long run" by "in the long term" (*en el largo plaza*); Sullivan (2010) uses "long-term proportion"
- replace "longest run of heads" by "largest number of successive [consecutive] heads" (*el mayor número de caras sucesivas*)

Also,

- replace "face of the coin" by "side of the coin" (to avoid confusion with *cara*[heads])

Another reason to distinguish similar-sounding phrases

- "long run" and "longest run"
- Mean, median, mode: Lesser & Winsor (2009) & CLASS survey

Pedagogical discussion

- Visuals (e.g., the sequence of flips, the bar chart of flips accumulated) help!

nlvm.usu.edu/es/nav/vlibrary.html

eduteka.org/MI/master/interactivate/

http://isi.cbs.nl/glossary/

What are your questions …or suggestions?

- Our ELL work and URLs of the resources:
  www.math.utep.edu/Faculty/lesser/ELL.html

- Contact us:
  Lesser@utep.edu or
  awagler2@utep.edu