

31 August 2012

Dear Professor Schield:

Thanks for your kind remarks for which I am flattered. Not sure if I deserve them. As for how I arrived at the Geometrical Interpretation of Simpson's Paradox, it actually happened quite naturally as follows:



In the early 1980s, I was working on graphical methods in Geometrical Optics and arrived at the main equations of geometrical optics graphically where the distances were represented by lines. The results were published in the Archimedean Journal *Eureka* [1]. At that time geometrical methods were also used to illustrate inequalities among the various means between two quantities. Several articles appeared in *Mathematics Teacher*, including one of my own [2].

A few year later, in the mid 1980s, the trapezoidal representation of the weighted average was published in *College Mathematical Journal* [3], which was followed by an article on Simpson's Paradox in the same journal [4]. I encountered an example of this rare phenomenon in professional Basketball [5]. The geometrical Representation of the "paradox" followed naturally from the weighted mean representation [3].

The method is largely based on the Geometrical Representation of the Weighted Mean by Hoehn [3]. The weighted mean between two numbers  $a$  and  $b$  ( $b > a$ ) depended largely on the weights and lay on a straight line from  $a$  to  $b$ . The weighted mean, therefore, could approach either number as closely as possible. Given a second set of two numbers  $c$  and  $d$  with  $d > c$ , Simpson's Paradox can be portrayed when  $d > a$ .

Geometrical methods and graphical constructions are always easy to visualize and comprehend. In fact, before the advent of Calculus, such methods were widely used to derive results. Now-a-days, algebraic equations are preferred because of their accuracy. Nevertheless, geometrical methods remain quite useful for illustrative and instructional purposes.

With best regards,  
Arjun.

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[1] A. Tan, Geometrical methods in geometrical optics, *Eureka*, **43**, 31 (1983).

[2] A. Tan, Mean constructions, *Math. Teach.*, **75**, 648 (1982).

[3] L. Hoehn, A geometrical interpretation of the weighted mean, *Coll. Math. J.*, **15**, 135 (1984).

[4] T.R. Knapp, Instances of Simpson's paradox, *Coll. Math. J.*, **16**, 209 (1985). Copy at [http://mathdl.maa.org/images/cms\\_upload/0746834219623.di020717.02p0042n.pdf](http://mathdl.maa.org/images/cms_upload/0746834219623.di020717.02p0042n.pdf)

[5] A. Tan, Letter to the Editor, *Basketball Digest*, **13(6)**, 6 (1986).