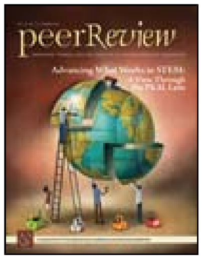

Summer 2011, Vol. 13, No. 3

Twenty-First-Century Quantitative Education: Beyond Content

By Shannon W. Dingman, assistant professor of mathematics, University of Arkansas, and
Bernard L. Madison, professor of mathematics, University of Arkansas

With the explosion of information and instant communication that is now available to the public, a statement attributed to Bell Laboratories mathematician Henry Pollak comes to mind. As computers became more powerful and ubiquitous in the latter part of the twentieth century, Pollak observed, “With technology, some mathematics becomes more important, some mathematics becomes less important, and some mathematics becomes possible.” As computers, the Internet, and Dick Tracy-like communication have immersed society in an environment alien to that in which many of us were educated, various analogs to the Pollak statement apply to different aspects of the educational landscape—that is, some things are more important, some less, and some now possible. This is especially true in the general education sector of mathematics and science education, where we work to move college students toward sound and effective quantitative reasoning (QR). How should quantitative education—and really, education as a whole—evolve to reflect the growing capabilities and demands of life in the twenty-first century?

New Possibilities

Any list of specifics made possible by the technological advances and sociological changes of recent years risks being out-of-date and incomplete within a few months. However, new possibilities affect quantitative education and how we can work to enhance student abilities to make sense of and effectively use the wealth of information around them. Some involve visualization, such as graphical representations of large data sets and geometric modeling. Others result from the almost instantaneous availability of information and questions surrounding validity of that information. This environment opens up new avenues for investigating and conjecturing, for allowing the curious to explore and reason, and for more complex real-life problems to be analyzed and understood. Yet they also place new demands on the explorer concerning the challenges of possibly accessing incorrect or misleading information. These factors, therefore, increase the demands for sound QR.

These new possibilities have influenced the creation and implementation of a QR course we teach to hundreds of arts and humanities students each semester at the University of Arkansas. Many of these students are quantitatively phobic and are averse to technology, save that involved in rapid communication (e.g., texting and e-mailing) and retrieval of information (e.g., through Google searches). This QR course has developed over the past seven years in the dizzying environment of changing technology. For curricular materials, we use media articles as the prompts for investigation. In particular, we began a decade ago with newsprint as the course first emerged. Since, we have found that few current students use newspapers and magazines as primary sources of information, but rather turn to the Internet as a guide. Nonetheless, regardless of the delivery medium, public media still chronicle the everyday world of our students, and they find QR in this everyday world at the same time interesting and challenging. Upon entering our course, the most serious weakness we have observed in many students' mathematical competency is what was termed "productive disposition" in the National Research Council Study Report, *Adding It Up* (Kilpatrick, Swafford, and Findell 2001, 31). As described there, "Productive disposition refers to the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that the steady effects of mathematics pays off, and to see oneself as an effective user of mathematics."

This weakness no doubt results from flaws in our system of mathematics education, and it is now both possible and critically important that this weakness be corrected. Quantitative reasoning in today's US society is no luxury or elective; it is an essential!

There are several aspects of our QR course that we believe adapt strongly to the current educational environment. First and foremost, the course is not organized by mathematical topics or the development of mathematical content, but rather is driven by quantitative societal issues reflected in public media (e.g., fuel efficiency, the national debt, credit card payments). Additionally, the course caters to student interests and current events, and provides a venue for continued practice beyond both the course and formalized schooling. One facet that allows students' interests to emerge is News-of-the-Day, a course component where students bring to class media articles with quantitative content to present and explain. This, of course, leads the class to unplanned and often unfamiliar areas of discussion (including for the instructor). At times we need information, and in some classes a student volunteer becomes the designated "web surfer," using a laptop or smartphone to provide key information such as definitions, populations, or other required data. In this pedagogical model, the instructor feels the shift away from being a dispenser of knowledge into being a moderator—clearly no longer the sage on the stage. This new role is a major change for faculty—from the fount of knowledge to the adjudicator of reasoning—and requires according pedagogical adjustments.

More Important and Less Important

Traditional content areas of mathematics—real and complex numbers, algebra, geometry, calculus, and more—dominated our educational experiences, and continue to dominate K–12 and collegiate mathematics. The power of this content has not diminished, but different aspects of it have become more important in QR education. Traditional educational practices have made students wary of and unprepared for dealing with the often fuzzy and ill-defined, yet very real, problems of their contemporary surroundings. Our QR students show very weak understanding of using their knowledge of school mathematics to solve real-life problems that emerge from the media articles. Contributing to this poor understanding is students' inadequate recall of school mathematics, which is due to lack of practice—they have not used it and, therefore, many have lost it. Additionally, the contexts of the media articles are different from the contexts of the application problems in school. These points argue for more relevant and varied contexts for applications in school and for more coordination of education in the various disciplines. QR contexts cover the spectrum of human activity—economics, health, politics, sociology, art, as well as science and engineering. The utility of developing and using interdisciplinary units to enhance student QR skills is obvious—the challenge is carrying this out. Over the past three summers, we have worked with grades 7–12 mathematics and science teachers to help them better understand QR and to develop investigations involving media articles that can be used in their classes. This cooperative effort aims to strengthen student QR abilities prior to college, and more is needed at both the K–12 and collegiate levels.

QR in everyday life is heavy on proportional reasoning, for example, to understand the quantities one encounters. "Just how big is this number?" "How can we know?" School algebra is of little use. Geometry will not help. This is not the mathematics of Euler and Euclid—it is the quantitative environment of the twenty-first century. How can we make sense of the size of the annual US military budget of $700 billion? How does it compare to the military spending of countries around the world? Just how large a quantity is the US national debt of $14 trillion? It is actually somewhat less than the current gross domestic product of the United States. How is this analogous to a person having a debt of one year's income? These questions are very much part of US public discourse at the moment, and many similar questions arise regularly in political and social arenas. These questions are important, yet the ambiguity involved in formulating answers requires flexibility in student thinking.

Whereas traditional mathematics spends considerable time in producing and manipulating representations, too little time is spent making sense of these representations. Yet today, rote procedures are less important because often they can be performed by technology or have no broad application. For example, after seeing the development of the formulas for and the connections between combinations and permutations, our students rely on their calculators for computing these counts. However, knowing the limitations of technology and what to do to push beyond those limits is important. For example, in calculating the probability that no two people among forty have a common birthday, students produce a quotient with a denominator and a numerator that will overflow many hand-held calculators, but rewriting the probability quotient as a product of forty quotients will push beyond this limitation. Summing the results of a daily compounding of interest in an installment savings problem can also exceed calculators' capabilities, giving reasons to develop the closed sum of a geometric series while recording where it came from. When there is a clear and present reason to use algebra, even our math-phobic students appreciate the effect. Traditional high school and nonmajor mathematics courses generally focus on calculation and manipulation of mathematical representations (functions, equations, expressions). Of course, this is still important, and regardless of the fact that much of this can be done by technology, understanding how it is done remains important. However, QR education (and many other learning outcomes) requires that we broaden teaching to include competencies such as interpretation of information and data, developing and evaluating assumptions, conducting analysis and synthesis of solutions to make sound judgments and conclusions, and communicating one's thoughts in an organized and coherent manner.

The Messy World of Realism

The complexity and messiness of real-life quantitative situations tax one's perseverance, disciplinary knowledge, and investigative habits. Students (and everyone) need to develop dispositions toward questioning and investigating. Knowing what to do when one does not know what to do is critical. Finding information is a breeze, but knowing if it is trustworthy has become a whole new ballgame in recent years. The professor and the textbook were trusted sources and remain so, but many other sources present themselves in classes such as our QR course. How does one know if information from Internet sources is reliable? One major criterion for trustworthiness we urge our students to utilize concerns the consistency of information with what they know. This opens up a whole new area of need because this criterion depends on what one knows. We refer to this knowledge as personal quantitative benchmarks. Sometimes these are as simple as knowing the approximate population of the US. However, sometimes the benchmark may be more complex—for example, knowing that more frequent compounding of interest on a savings account will increase the balance. In his 2008 book *Stat-Spotting*, Joel Best lists a few quantitative benchmarks needed to understand US social statistics. Three basic ones are the US population, and the annual number of births and deaths in the US. Building up an inventory of personal quantitative benchmarks promotes further investigation and evaluation of information, leading to the habit of mind that is quantitative literacy. Habits are developed by continued practice, making provision of venues for practice beyond the classroom and school critically important in various areas of reasoning and rationalization.

Connected to the issue of quantitative benchmarks and validity of information is the issue of quick and efficient evaluation of information to decide if further investigation or vetting is necessary. While reading quantitative arguments or assertions in public media, one needs to be able to detect when arguments or assertions seem correct or flawed. Detection can depend again on what one knows, but it can also result from approximate calculations involving the quantities in the argument or assertions. Grabbing a calculator or a pencil is often inconvenient or impossible. Thus one relies on mental calculations, estimation, and ad hoc reasoning. One of the bad results of calculators in schools is an overreliance for even the simplest calculations, producing students unpracticed at mental arithmetic. Some students seem inclined toward on-the-fly thinking, and some profess that it is because they believe they are avoiding work. In fact, mental calculation can lead to sound examples of QR. For example, one of our students illustrated on-the-fly thinking in answering a question regarding the amount of the 2001 US federal budget. This question stemmed from a statement by economist and columnist Paul Krugman that $1 billion per month (the estimated cost of the war on terrorism) was about one-half of one percent of the annual federal budget (in 2001). “Well,” said the student, “one half of one percent is $12 billion, so one percent is $24 billion, and 100 percent is $2,400 billion, or $2.4 trillion.”

The Complex Learning Outcomes Landscape

Moving away from channeled disciplinary education to cross-disciplinary education with increased attention to reasoning and other cognitive processes has prompted considerable thought to a structure for learning outcomes. One example provides some hint of the complexity of possible landscapes. The intricacy of these learning outcomes structures reflects the challenges of mathematics and science education, of all education in the twenty-first century. AAC&U’s Valid Assessment of Learning in Undergraduate Education (VALUE) project provides rubrics to evaluate achievement of learning outcomes, including intellectual and practical skills and areas of personal and social responsibility and integrative and applied learning. These include inquiry and analysis, critical thinking, written communications, and quantitative literacy, among others. The quantitative literacy VALUE rubric contains six core competency areas—interpretation, representation, calculation, application/analysis, assumptions, and communication—and four performance levels for each competency area.

The rubrics are intended for institutional-level use in evaluating and discussing student learning. We used the quantitative literacy VALUE rubric, however, as a springboard for thinking about assessing students’ QR. Because messy and complex QR problems lead to complicated assessment of student learning, accurately scoring student responses is both more difficult and more important than ever before. Multiple-choice tests are rarely an option here. Assessing QR calls for attention to reasoning structure and scoring rubrics that are more complex than those used to score simple calculations, which comprised much of what we scored in the past. Along with colleagues Stuart Boersma and Caren Diefenderfer, we modified the VALUE rubric to one that we successfully used to score individual student work in answering study questions from our QR casebook used in our QR course. The major value of the rubric, as we discovered, was not just in the consistent scoring it provided, but also in the assistance it provided for preparing course materials and assessment tasks and for helping to guide student thought processes in QR.

Conclusion

Being an informed and productive citizen in the twenty-first century is more complicated than ever before, and the educational experiences we offer to students need to reflect this complicated world in which they operate. Traditional education has long centered on content to drive learning, with the surrounding skills and processes being developed from student work with the content. However, with continuing evidence that students are not gaining the skills they need and with technology providing greater access to working with content, we must consider how traditional education can better support the development of these skills and produce students better equipped for citizenship and the workplace. This is not to suggest that content should be ignored; in fact, we must work to ensure that students possess both the knowledge *and* skills desired of a learned citizenry.

Our work in QR education is in a small corner of this broad educational picture, but we believe our experiences are meaningful across much of the landscape. Indeed, one component of the educational system that has become more important is synergistic teaching and learning. The same processes we promote in QR should be the processes in physics, chemistry, economics, and biology. The QR core competencies—as we use them—of interpretation, representation, calculation, analysis and synthesis, assumptions, and communication have closely related competencies in all subjects. These core competencies can be used to examine whether the learning experiences provided to students truly capture the nature and breadth of skills needed to be successful in the twenty-first century. Our world is ever changing, and it is therefore vital that the education provided to students evolves as well in order to develop citizens that are well prepared for the world they encounter. Much of our work is reported in *Numeracy*, the journal of the interdisciplinary National Numeracy Network (NNN), where other resources for assessing QR can be found.

Acknowledgements

Development of the quantitative reasoning course discussed in this article has been supported by a National Science Foundation grant (DUE-0715039) and by two colleagues, Caren Diefenderfer at Hollins University and Stuart Boersma at Central Washington University. Similar courses are being taught at Hollins University and Central Washington University.

References

Association of American Colleges and Universities. 2009. “Assessing Learning Outcomes: Lessons from AAC&U’s VALUE Project.” *Peer Review* 11 (1).

Best, J. 2008. *Stat-Spotting: A Field Guide to Identifying Dubious Data.* Berkley, CA: University of California Press.

Boersma, S., C. Diefenderfer, S. Dingman, and B. Madison, B. 2011. “Quantitative Reasoning in the Contemporary World, 3:Assessing Student Learning.” *Numeracy* 4: 2, article 8. http://services. bepress. com/numeracy/vol4/iss2/art8

Kilpatrick, J., J. Swafford, and B. Findell (eds.). 2001. *Adding It Up: Helping Children Learn Mathematics*. Washington, DC: National Academies Press.

Madison, B., S. Boersma, C. Diefenderfer, and S. Dingman. 2009. *Case Studies for Quantitative Reasoning: A Casebook of Media Articles*, 2nd Edition. New York, NY: Pearson Custom Publishing.