Randomness & Chance

Statistical Literacy 2009

Chapter 7 Overview

by

Milo Schield

www.StatLit.org/...

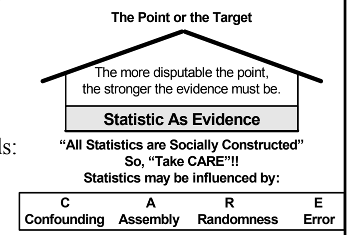
pdf/2009StatLitTextHandoutCh7.pdf

audio/2009StatLitTextHandoutCh7.mp3

Ch 1. Review

Statistics are generally used as evidence to support an argument.

The influences on a statistic are of four kinds: Context, Assembly, Randomness or Error.



Randomness

1. How do random excesses correct? Offset or dilution?

2. How can we use chance to estimate and adjust?
- Truthful Answers on Sensitive Issues using Chance
- Adjusting for Guessing using Chance
- Estimating Population Size Using Chance

3. How to estimate survey sampling error @ 95% confidence.
- 95% Margin of Error for subgroups
- Sample size needed to achieve desired accuracy.

4. What is *statistical significance*? Calculation and meaning.

Overcoming Excess

A fair coin has been giving an excess of 'heads'.
Should you bet 'tails'? Must coin offset to get 50-50?

A roulette wheel has been giving an excess of 'Red'.
Should you bet 'black'? Must wheel offset the 'red'?

No! A coin and a wheel have no memory.
They have no power to offset. Offset is impossible!

So how do they return to 50-50?
By dilution. Just doing 50-50 will dilute any excess.

Truthful Answers on Sensitive Issues

Sensitive question: Have you done something bad?
Assign people to two groups: Truthful and "Yes-only"

	A	B	C	D
1		Randomly assigned		
2		Say "Yes"	Tell the Truth	ALL
3	Answered "No"	0	+D3	+D5-D4
4	Answered "Yes"	+B5	+C5-C3	**# Saying Yes**
5	ALL	+D5/2	+D5/2	**Total #**

Fraction of truthful who say Yes: (Yes – N/2) / (N/2)
This assumes random assignment gives a 50-50 split.
Never check on this. Doing so breaks anonymity.

Adjust for Guessing

N multiple-choice questions with k possible answers.
Decision: Mark answer or leave blank.

If one guesses: 1 chance in k of correct answer.
Pure chance: N/k right; N-N/k = N[(k-1)/k] wrong.

To minimize guessing, make net score equal to zero
Subtract 1/(k-1) points for each wrong answer.
N/k right. Subtract {N[(k-1)/k]}[1/(k-1)] = N/k.

Rule: Guess only if chance of right answer > 1/(k-1)

VoD 2009 Statistical Literacy Textbook Handout: Ch 7 7

Estimate hard-to-count population

Examples: Fish in a lake or uncounted in a census. This method of estimating the population (N), involves capture-recapture: taking two random samples at different times.

1. Count and mark those in the first sample. Call the count n.
2. At a later time, take a second *random* sample of size s. Find that m of these s are marked from the first sample.

3.Results:
- Proportional reasoning: If $m/s = n/N$ then N = n(s/m).
- Fractional method: If $m/n = s/N$ = p, then N = n/p.

Example: 100 tagged in 1st group. 2% of 2nd random catch had tags. Estimated population = N = 100/0.02 = 5,000.

VoD 2009 Statistical Literacy Textbook Handout: Ch 7 8

Sampling Error

Sample means are distributed around population mean.
Sample error: Sample mean minus population mean.

Margin of error:
1) is expected sample error with 95% confidence.
2) decreases as sample size (n) increases.
3) is proportional to 1/square root of n.
Quadruple sample to cut error in half.
4) is independent of size of population.
Counter-intuitive. Key is randomness: well-mixed.
To see if coffee is too weak/strong, need just one sip.
2 is expected; 3 is a small surprise; 4 is a big surprise.

VoD 2009 Statistical Literacy Textbook Handout: Ch 7 9

Margin of Error Proportions

The exact 95% margin of error for a proportion is:
1.96 * Sqrt[p*(1-p)/n] where p is sample proportion.

The conservative 95% margin of error is:
1/Sqrt(n) where n is the sample size.

This conservative value is typically reported in surveys. It always includes the entire group and it always assumes p=50% so it gives the largest interval.

Confidence interval of a proportion = p ± ME

VoD 2009 Statistical Literacy Textbook Handout: Ch 7 10

Survey Margin of Error For Subgroups

Most polls show the Survey 95% margin of error.

This is the most-conservative margin or Error for the entire survey. Assumes p ~ 50%.

Let N be size of random sample; n = size of subgroup.
Let F = fraction of group that is in a subgroup = (n/N).

Result:
Subgroup ME = Survey ME √(N/n) = Survey ME/Sqrt(F)

If F = 9%, then 95% subgroup ME = 95% Survey ME/0.3

VoD 2009 Statistical Literacy Textbook Handout: Ch 7 11

Confidence Intervals: Measurements

Conf. Interval: Sample Mean ± ME.
95% Margin of Error ≈ 2s/sqrt(n)

Randomly select 100 students (n). Suppose they average 8 hours working per week with a standard deviation (s) of 5 hours.

1) What is the estimated population mean? Answer: 8 hrs.
2) What is the 95% margin of error for the average time working/week? A. 1 hr: 95% ME = 2s/Sqrt(n)=2*5/Sqrt(100)
3) What is the upper limit of 95% confidence interval for average time working? Answer. 9 hrs: 8 hrs + 1 hr
4) What is the lower limit of 95% confidence interval for average time working? Answer. 7 hrs: 8 hrs - 1 hr

VoD 2009 Statistical Literacy Textbook Handout: Ch 7 12

Required Sample Sizes [Not in this text yet]

Suppose allowable Margin of Error = E

1) Proportions (most conservative): ME = 1/sqrt(n) = E
So n = $(1/E)^2$ Doubling E quadruples n
Example: If E = 0.02 then n = $(1/0.02)^2 = 50^2$ = 2,500.

2) Proportions (Exact). ME = 2*Sqrt[p(1-p)/n] = E.
n = [p(1-p)] $(2/E)^2$. IF E = 0.02, p = 0.2; n = 1,600.

3) Measures: ME = 2s/sqrt(n) = E
n = [E/(2s)]^2. If E = 100 and s = 1, then n = 2,500.

V0D 2009 Statistical Literacy Textbook Handout: Ch 7 13

Statistical Significance: Unlikely

Statistical significance (statistically significant):
Unlikely (< 1 chance in 20) if due just to chance
- Coins: 5 heads in a row (1 chance in 32)
- Cards: Dealt two Aces ((4/52)(3/51) = 0.038 < 1/20

Stat. Significant => More likely due to non-chance!

Statistically insignificant (not statistically significant)
Likely or *not unlikely* (> 1 chance in 20) if due to chance
- Coins: 4 heads in a row (1 chance in 16).
- Cards: Dealt one Ace (4/52 or 1 chance in 13)

V0D 2009 Statistical Literacy Textbook Handout: Ch 7 14

Statistical Significance & Confidence Intervals

Statistically insignificant (not statistically significant)
Likely (> 1 chance in 20) if due just to chance
Difference: Two 95% confidence intervals do overlap

Statistical significance (statistically significant)
Unlikely (< 1 chance in 20) if due just to chance
Difference: Two 95% confidence intervals do not overlap
Extremely conservative. Sufficient but not necessary.

Suppose the percentage who are for Obama was 47% last month and 51% this month – with a 2 point margin of error. Is this change statistically significant? A. No!

V0D 2009 Statistical Literacy Textbook Handout: Ch 7 15

Statistical Significance: Importance and Cause

Being statistically-significant
- Does NOT mean "important" or "note-worthy"!
- Does not mean "unlikely TO BE due to chance"

Being statistically significant
- Means "unlikely IF due just to chance"
- Does support the claim that the outcome is more likely to be due to something other than chance.

V0D 2009 Statistical Literacy Textbook Handout: Ch 7 16

Statistical Significance: Not Due to Chance?

Q. Does statistical-significance ever mean unlikely *to be due* to chance?

A. Depends on whether one allows these statements
Yes! If the chance the alternate is true is more than 50%.
No! If the chance the alternate is true is less than 50%.

Example: In an ESP experiment, a subject's choices compared to chance were statistically-significant.

Analysis. Since the chance that ESP is real is much less than 1%, this result is still likely *to be due* to chance.

V0D 2009 Statistical Literacy Textbook Handout: Ch 7 17

Statistical Significance: Necessary?

Q. Is statistical-significance necessary?

A. Sometimes

Yes! In clinical trials of new drugs or medical procedures where highest standards are required. C.f., criminal trials with presumption of innocence and requirement of 'guilt beyond reasonable doubt."

No! In polls there is no reason to assume there is no difference in the popularity of two candidates (or in the popularity of a single candidate over time). C.f., civil trials – no presumption; requirement is "preponderance of evidence" (More likely than not)

V0D 2009 Statistical Literacy Textbook Handout: Ch 7 18

Statistical Insignificance Explanations

Q. What explains statistical insignificance?

A. Two kinds of explanations:

1. Nothing real; no real difference; just coincidence.
2. Real difference but small so it is indistinguishable from chance/noise. Might be seen in a larger sample.

Cannot conclude there is no real difference (#1)!!!

"No difference between samples" does not mean "no difference between populations."

V0D 2009 Statistical Literacy Textbook Handout: Ch 7 19

Summary: Randomness

Sampling error is often overlooked as an influence on statistics or statistical associations.

A 95% confidence interval includes the population parameter (is right) 95% of the time.

A **statistically-significant association** is not necessarily an important association.

A **statistically In-significant association** may be pure coincidence or it may be a real association.