Quantitative Literacy in America: What Kind to How Much and Beyond

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About four years ago I was invited by AAC&U's *Peer Review* to write an essay on quantitative literacy for a special issue on the topic by that academic journal. As I recall, the initial request was for an essay that addressed the objections of mathematicians to QL as an academic pursuit, but, as is often the case, the essay eventually became something else. Often I have expected to write or speak on a particular topic, but when it came time to produce I found the topic uninspiring or at least I was uninspired. That, in fact, may be the case here today – I will leave that for you to decide. I told Geoffrey that I would talk about some history of QL, or numeracy, and relate that to life in 21st century America, something like I did at the symposium talk I gave last November to AMATYC. I still plan to do that but with a slightly different approach.

Getting back to the *Peer Review* article, which was titled, Two Mathematics – Ever the Twain Shall Meet. SLIDE 2 Here is how the article began, with a few asides for us today:

In his classic essay on the aesthetics of mathematics, *A Mathematician's Apology*, the prominent British mathematician G. H. Hardy (1940) identifies two mathematics: real mathematics and "trivial" (or useful) mathematics. Although writing from a rarefied point of view, Hardy aptly describes a disparity that has existed for centuries, one that persists in twenty-first-century America as a division between the rigorous mathematics that real mathematicians study, appreciate, and extend and the contextualized mathematics of everyday life.

The American historian Patricia Cohen confirms a similar division in colonial times, between commercial arithmetic and more sophisticated mathematics. Fearing that bourgeois lads would find mathematics too difficult, colonial and pre-colonial textbook writers attempted to strip arithmetic to its essentials. But, as Cohen (1982) points out, "in fact they cut it into incoherent bits and made it an arcane subject, almost impossible to learn" -- compare that to the school and college algebra of today. To make matters worse, arithmetic was suffused with commercial meanings, which caused those not destined to a life in commerce to learn no arithmetic at all. The endemic lack of arithmetic skill among tradesmen in the seventeenth and eighteenth centuries is indicated by the popularity of "ready reckoners," books containing page after page of tables showing multiples of unit costs of common commodities at a variety of prices – in some people's views, the 19th century version of the hand held calculator. You can draw amazingly parallel analogs in the applications of mathematics today in areas such as finance, public policy, and sociology. Thus the need for initiatives such as mathematics across the curriculum.

As American democracy has developed, the quantitative demands on its citizens have grown; indeed, largely driven by the power of computers to amass and analyze data, these demands have exploded over the past two decades. The ability to deal with the quantitative demands of everyday life is what we call quantitative literacy (QL for short). Robert Orrill has described QL as a cultural field where language and quantitative constructs merge and are no longer one or the other, reflecting the continued suffusion of arithmetic with meanings from societal contexts.

Although no one has directly refuted my two mathematics thesis, one can certainly find evidence to the contrary. One can cite numerous examples – mostly unanticipated by the mathematics development – of applications of mathematics: Kepler's laws of planetary motion, Fourier analysis, cryptology, coding theory, game theory, quantum theory, and many others. Why is mathematics "unreasonably effective," as physicist Eugene Wigner stated? Harvard mathematician Andrew Gleason explains it by noting that mathematics is the science of order, finding order in very complex situations, and the world is a complex situation in which there is a great deal of order. Physicist Arthur Jaffe explains another reason: that mathematics takes its inspiration from patterns in nature and these lessons from nature continue to serve well when we explore other phenomena in nature.

Of course, almost all of these application areas cited here and any that get notice involve advanced mathematics well beyond what ordinary college graduates would encounter. It is the mathematics of school and the early years of college that remains apart from the quantitative reasoning needs of ordinary citizens. US school and college mathematics has not changed very much in the past century, in spite of the several efforts of reform. That college mathematics has not changed is quite amazing considering the vast sea changes in students and in society. The only significant change in US undergraduate education ever was the paradigm shift from the classical curriculum to the system of majors and general education in the first part of the 20th century, and the mathematics of college did not change very much even during this shift. The algebra teacher of 1850 would be reasonably at home with the algebra course of today, and that algebra course, now spread across multiple years in high school and college, remains the staple offering of mathematics for general education. School and college mathematics is largely dictated by the needs of developing engineers, scientists, and mathematicians, with only a bow to general education as an expected consequence.

Well, my thesis here today is that that expectation – of solid quantitative general education – is not being met, has not been for some time, and is certainly not meeting the quantitative reasoning demands of contemporary US society. There is a serious mismatch between the quantitative demands that US society places on its citizens and the quantitative education made available. If US society is to survive on the basis of an informed electorate, then not only must the mathematics of school and college redirect much of its effort, but mathematics must cooperate with other disciplines to a much larger extent than it does now. Therefore, the need for initiatives such as mathematics across the curriculum and quantitative reasoning.

What I see as the potential big accomplishment of mathematics across the curriculum and quantitative reasoning or literacy initiatives is the bridging this gap between the two mathematics – formal school and college mathematics and the mathematical reasoning necessary for an informed life in 21st century America.

As Winston Churchill said, the farther back we look, the clearer we can see ahead. To outline the problem more clearly, let's go back a few centuries – ten or so. As we move forward in time I will expand on some of the areas I have mentioned.

SLIDE 3 The earliest mathematics was used for practical purposes – counting, measuring, surveying, and building. But mathematics had a special appeal to intellectuals – Plato, Aristotle, Euclid, and Archimedes, for examples. Plato was quite an idealist and abstract thinker. He advocated for one to "…turn away from the material world because it is always becoming and never is and turn toward that which always is and has no becoming… To absolute beauty, goodness, and righteousness, and to the ideal triangle, square, and circle, to abstractions that exist independent of the material world." Even today, Platonist refers to mathematicians who view the abstract generality of mathematics as central to its power and understanding. This view of mathematics as divorced from reality led to a separation that historian Alfred Crosby dates from about the 5th century. SLIDE 4 The western world – mostly Europe remained un-quantified until about the 13th century when the needs of keeping time, measuring quantities for commerce, book-keeping, navigation, and warfare motivated quantification. A very nice account of this period, 1200 – 1600, is given in *The Measure of Reality* by Crosby… detailing the movement from qualitas to quantitas, from St. Thomas Aquinas to Galileo – from what kind to how much.

In the1650s, a few years after Galileo's death, a seemingly insignificant event triggered what has become an enormous part of modern society. A self-styled gambler and philosopher, Chevalier de Mere, wanted to know how to distribute the winnings in an unfinished game of chance. The problem was posed to Blaise Pascal who solved it with help from Pierre de Fermat and the theory of probability emerged from the collaboration.

The account of this event and much more are described in another marvelous history book, *Against the Gods*, SLIDE 5 by Peter Bernstein. Of course, the theory of probability provides the basis for the quantification of risk, which we know to be fundamental in today's economies and other aspects of life.

About 300 years after the development of probability, in 1953, economic Nobelist Kenneth Arrow used tools of quantification to articulate a world in which every potential outcome would have a predictable price tag. This idealized "complete market" is a direct albeit distant descendent of the nobleman's unfinished games of chance.

This "complete market," which now dominates the US economy – insurance of all kinds, stock market derivatives, lotteries, hedge funds, and futures markets – is the descendent of a much simpler and smaller market of America in colonial times. A third marvelous history book by Pat Cohen SLIDE 6 details this period from the point of view of numeracy.

Early on the American attitude and inclination was toward counting and calculating, and this inclination has only increased over the past two centuries. You can read here what two European travelers in America in 1830 observed about the penchant the Americans had for counting, calculating, and competing. Several features of US society significantly increase the quantitative demands on all of us. Others noted this American penchant for quantification. SLIDE 7.

Matthew Arnold, English poet and cultural critic, in citing a fellow countryman in 1884 warned Americans about the more is better culture. As Bob Orrill put this in his Wingspread essay on QL, "In plain speech, Arnold means that all this talk of abundance is tiresome stuff." Even a century ago, the American penchant for quantifying was seen as wrongheaded. There was worry that qualitas to quantitas had gone too far.

This might be my subtext for today – Have we gone too far in quantifying society? If we have not, can we educate our citizens so that they can understand these quantifications?

SLIDE 8: The early American contributors to counting & quantification were the issues of apportionment in the US and state and local governments… requiring a census the results of which serve to describe our country in many quantitative ways. The census has not been without controversy, having encountered along the way demands for privacy, independence and bumped into issues such as mental health and slavery that intensified the debates.

The American penchant for individual freedoms and economic competition with minimal economic safety nets has created significant demands for quantitative reasoning for survival in this somewhat Darwinian society.

In recent years, deregulation and privatization have greatly complicated decisions facing us. Just think about the extent – airline deregulation – remember when fares were regulated and compare that to the jungle of fares now. Banking deregulation has created thousands of new banks, many with their own credit cards and savings & loan plans. Consider the demands created by the partial privatization of Medicare prescription drug insurance or the many retirement plan options that go under the banner of IRAs. If the social security system is partially privatized as was proposed before it got pushed aside by the war on terror then we will see hordes of options presented to us.

Churchill also said that we shape our buildings and thereafter they shape us. We have quantified US society and that quantification is now shaping us.

Numbers are everywhere – and numbers never lie! SLIDE 9: For example, *Time* now has a column in its weekly issues that invites one to draw conclusions from comparing two quantities – numbers with units, but often with different units that signals we have mathematical difficulty in making a direct comparison. Here is one example that accompanied a Time article about the 50th anniversary of the Little Rock, Arkansas, school desegregation crisis. SLIDE 10: The two quantities are percentages: (1) 89%: Percentage of white students who attend schools that are more than 50% white, and (2) 72%: Percentage of black students who attend schools that are more than 50% minority.

There is little doubt that US residents carry the world's heaviest QR burden, but US schools and colleges have failed to respond. We continue to teach mathematics and even probability and statistics without taking into account these quantitative reasoning needs. Let's look at some differences that perpetuate this separation into two mathematics – differences between formal school mathematics and QR or QL. (SLIDE 11)

School Mathematics	**QL or QR**
• Power in abstraction	• Real, metamorphic contexts
• Power in generality	• Specific, particular
• Some context dependency	• Heavy context dependency
• Society independent	• Society dependent
• Apolitical	• Political
• Methods & algorithms	• Ad hoc methods
• Well-defined problems	• Ill-defined problems
• Approximation	• Estimation is critical
• Heavily disciplinary	• Interdisciplinary
• Problem solutions	• Problem descriptions
• Few opportunities to practice	• Many practice opportunities
• Predictable	• Unpredictable

My mathematician colleagues have very mixed views of QL/QR. Some dismiss it as middle school mathematics and decry the lack of QL as a failure to learn mathematics. Most say that learning mathematics well will result in QL/QR. Perhaps that is true, but "learning mathematics well" is surely a soft spot in that argument. Let me illustrate: I teach a course that I developed and am now expanding with NSF's help to college students seeking the BA degree, mostly in the arts and humanities. The course has a college algebra prerequisite, so all these students either have credit in college algebra or have test scores that get them placed beyond college algebra. Here are two problems (SLIDE 12) that have stymied some of my better students. Of course they can solve these once they are coached a bit, but this illustrates the fragility of the usefulness of their algebra

$\frac{52}{k}=18$ Solve for k. This arose in a study of how the Dow Jones Industrial Average is calculated.

$x+0.06x=500$ Solve for x. This arises when one wants to know the value of a quantity that has increased by 6% and is now 500. This problem illustrates a conceptual leap from reducing or increasing a quantity by 6%.

I also lead our mathematics major seminar, a capstone experience for majors. There are about 1/3 BA students (most expecting to teach in high school) and 2/3 BS students (various plans including graduate school in mathematics or some related discipline) – 20-30 each year. It is reasonably easy to stump most of them with contextual problems such as instances of the false positive paradox or Simpson's paradox. The BA students have very limited knowledge about the connections of their undergraduate mathematics courses and school mathematics – analogous to all of them having very limited knowledge about the connections of their undergraduate mathematics courses to the world of everyday quantitative issues. I do not feel bad pointing out these weaknesses because I surely had them when I was a senior mathematics major --- and beyond.

In the course I have developed and am expanding with the help of others, notably Caren Diefenderfer (Hollins), Stuart Boersma (CWU), and Shannon Dingman (UAF), we have identified a dozen areas of QL situations. Here is a rough schematic of our approach with QL situation areas across the top and mathematical areas down the left. SLIDE 13:

	Using Numbers	Percent and Percent change	Units and Measurements	Rates of Change	Graphical Representations	Odds and Risk	Averages and Indices	Weather maps and measures	Using data (including tabular presentations)	Personal Finance	Voting and Apportionment	Estimation and Approximation
Ratio and proportionality	X	X	X	X	X	X	X	X	X	X	X	X
Counting	X		X			X			X	X	X	X
Deductive and Inductive Argument	X	X	X	X	X	X	X	X	X	X	X	X
Linear and Exponential Growth	X	X		X	X				X	X		X
Geometry	X		X									X
Probability	X	X	X			X		X	X	X		X
Statistics	X		X		X			X	X	X	X	X
Graphs	X	X	X	X	X				X	X		X

We believe that every problem has a context and that there are levels of difficulty driven mostly by the context – the reasoning – but not necessarily the level of the mathematics. This is a key point to remember:

The difficulty of interpreting and resolving a QL situation is not determined by the level of the mathematics or statistics involved. We have to get over that.

We teach this course using real world media articles – both continuous prose and discontinuous prose (graphs, charts, etc.) – so we have hundreds of QL situations that cross through every academic discipline. I want to show you a few from the graphics area where graphs students see in algebra class almost never occur in news media.

Graphics occur in political arguments. SLIDE 14: Here are two graphics that show the two political parties wanting to use different units – nominal dollars, constant dollars or percentage of GDP – to make a case in a argument. One interesting aspect of this is that the parties switch sides on which units are preferred in the two graphics. Measuring in different units is a major source of QR situations. Here is another example SLIDE 15 where there are two different graphs pertaining to a tax cut – a Democratic and a Republican view. The independent variables are the same – the various income intervals – but the two different variables are being used for the dependent variable – one uses the percentage that taxes would be reduced for an income interval while the other uses how much of the tax cut in dollars would accrue to people in the income interval. That change gives very different visualizations of the results of the tax cut. Back for a moment to the GOP disputes – our case study on these two graphics shows the risks of venturing out of our academic silos – we got taken to task by a physicist advisor and an economics advisor – physics for comparing kilograms and pounds and the economist for our use of real dollars – mistakenly labeled in the AP graphic.

Graphs occur to explain some national policy decision. SLIDE 16: Here is one of the US troop surge in 2007 in Iraq. Quite frankly, I see no helpful information given here. It seems to be quantitatively useless and makes little sense.

Unusual interesting & innovative graphs crop up from time to time. Here SLIDE 17 is one brought to my class by a student. I do not know the source. The interesting feature of this graphic is that it is a bar graph on a circle that represents the face of an analog clock. The bars represent the number of students in a plaza at each of 24 hours – visually very helpful.

Sometimes graphs describe some social or national circumstance of critical importance. Here is one sent to me by Joel Best that purportedly shows the recidivism rates for various crimes. SLIDE 18: The graph is basically a picture because the apparent appeal to a pie chart is not at all accurate – note strangeness of approx 1/5 being 3/5 or ¾. The use of "expanding circles" rather than bar graphs in becoming more common. Here are two – both from the *New York Times*. One is the depiction of increasing (ballooning? health care expenditures. SLIDE 19: The other SLIDE 20 uses circles to represent amounts in this graphic about non-US contributions to the current Democratic presidential candidates.

Here SLIDE 21 are two news articles reporting the same information about the fall enrollment at the University of Arkansas in 2004. The news is that the enrollment increased over that of fall 2003, and one of the articles reports that the graduation rate had increased also. There are several lessons here. One is the use of a line graph to represent seven data points. This is particularly troublesome since the enrollments between the fall data points in all likelihood decreased, so the line is not useful for estimation of, say, the spring semester enrollments. A second issue is the headline and subhead on the one article – by the way, usually written by someone other than the reporter -- get the news items reversed. The graduation rate increased, not the enrollment rate, and the enrollment increased, not the number of graduates (at least that is not given in the data). The other issue is the visual exaggeration of the increase by shifting the scale on the vertical axis so that the interval begins at 14000 rather than 0 – as in the bar graph. The rate of change of a quantity is very often confused with the quantity itself.

The business page has had graphs for many years, but only recently have I seen two dimensional complex graphics like this one SLIDE 22 from the New York Times. The horizontal scale measures the return rate on stocks (in the software sector) for the week and the vertical scale measures the return for the year. The origin is not (0,0); rather, it is the return for the week and year on the S& P 500. And the size of the circular disk representing a company measure the capitalization (value) of the company. At least this graph comes with some instructions about what it means SLIDE 23 but these meanings are stated in quantitative stock market word such as slipping, leading, lagging, and improving. But SLIDE 24 shows that USA Today can one up the Times by adding color and a third dimension.

Enough – we could go on and on… There are examples of similar graphics and interpretations in many newspapers and magazines on a regular basis.

These examples and numerous other quantitative situations in everyday life points to some goals we should have for our students SLIDES 25 & 26.

Goals for our students

- Calculators with savvy
    - Estimation and approximation
    - Proportional reasoning

  - Compute using unknowns
- Attentive to units
  - Numbers are adjectives
- Readers and communicators with requisite precision
  - Difference between causation and correlation
  - Difference between coincidence and conspiracy
  - Read continuous and discontinuous text
    - Maps, charts, & graphs
    - Words and numbers
- Critical consumers of numbers (quantities)
  - Is it reasonable?
  - Is it unusual? How surprised should we be?
  - Is it correct?
  - Is it meaningful?
  - How was it determined?
- Careful and effective users of numbers
  - Quantitative arguments
  - Assessing risk
  - Measuring change

Conclusion
I should point out that there are some very thorny problems involved in defining and delineating the integrative QL that I have talked about. Psychologist and assessment expert Rich Shavelson talked about some of these at Wingspread – for example the three historical perspectives SLIDE 28 for defining QL or QR: psychometric (behavioral roots), cognitive (mental processes roots), and situative (social-contextual roots). Rich explained the differences with: psychometricians ask, “How fast will the car go?”; cognitive scientists ask, “How does the engine make the car go fast”; and situativists ask, “How is the car used in this particular culture?” You might want to think about how school mathematics fits into these three perspectives. I hope you realize from what I have said today that my own views are heavily situative. We need to understand this psychological situation if we are to make the case for QR widely. But not today… we are out of time.

I could go on for a good while showing you examples of situative QR issues that face US residents daily. If the US is to survive with its agile economy fueled by private enterprise and consumerism, freedoms to amass wealth, to become destitute, and to be governed by an informed electorate, then we must achieve stronger reasoning skills – quantitative reasoning skills – across our population. We advertise college as preparation for getting a job; yet the Bureau of Labor Statistics for the US Department of Labor estimates that only about ¼ of the jobs available in 2014 will require any college. That’s a few percentage points below what it was in 2004. But every citizen will still need to make decisions about health, public policies, personal finance, environmental protection, education, and entertainment – and probably a few percentage points above 2004. US society is sure to become more complex quantitatively by 2014, and college can help. The argument we should be making for college is that you need it to understand and enjoy the increasingly Darwinian society in which you live. But, to be faithful to that argument, college needs to change, and, in particular the way we treat quantitative literacy and reasoning needs to change. Your efforts on behalf of mathematics across the curriculum and QL/QR are encouraging steps in that direction. But they are small steps, and there is much work to be done, both in developing programs and in research on how QL is achieved.

The educational issues are very difficult – the system has enormous inertia. We must emerge from our disciplinary silos and work together – the stakes are very high --- in my view the survival of our democratic processes.