Assessing the Quantitative Literacy of Students at a Large Public Research University[1]

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Introduction: Examining the Need for Quantitative Literacy

Steen (2001) claims that, “most U.S. students leave high school with quantitative skills far below what they need to live well in today’s society; business laments the lack of technical and quantitative skills of prospective employees; and virtually every college finds that many students need remedial mathematics” (p. 1). Echoing the plea in A Nation at Risk (1983), Everybody Counts (1989) urged Americans to focus seriously on the underachievement of our students in mathematics.

Reforming mathematics teaching and learning through policy has been at the forefront of mathematics education agendas (e.g., NCTM, 1989, 2000; also see Porter et al., 1993; Webb, Heck, and Tate, 1996; Wu, 1996). Three significant questions addressed by the mathematics education reform that inform the discussion of quantitative literacy are: 1) **Who** should have access to mathematics? 2) **What** mathematics should all students learn? and 3) For what purpose, or **why**, should we teach mathematics? Through this wave of reform, mathematics educators have introduced a vision of mathematics “for all students” along with a commitment to prepare a new generation of workplace- and college-bound students (NCTM, 2000; NRC, 1989).

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This new vision of mathematics education (see NCTM 2000) places a substantial emphasis on learning mathematics with understanding, communicating mathematical ideas, and the applicability of mathematics to life.

Despite efforts for large-scale mathematics educational reform, our population still largely remains unprepared for professional and personal demands for quantitative skills (Hughes-Hallett, 2003). College faculty are faced with students whose mathematical preparation is diverse and who are often unable to retain and utilize their previous quantitative experiences, regardless of how substantial, meager, or non-existent (Steen, 2004). As it becomes clear that K-12 mathematics education does not prepare many for the ability to use mathematics to “make wise decisions at work and in public and private life” (Hughes-Hallett, 2003, p.91), the overall goals of mathematics education must expand to include the attainment of quantitative literacy.

A student’s ability to apply his or her mathematical knowledge in a real-world context is rarely assessed, despite its importance as a goal for mathematics education. As we begin to focus on preparing students to utilize their mathematical knowledge within and more importantly outside the classroom walls, we must not only assess the algebraic capabilities of incoming students, but their ability to utilize those mathematical skills in an everyday or professional capacity. This is the essence of quantitative literacy.

Background: Defining Quantitative Literacy at MSU

On January 20, 2005 the Michigan State University Quantitative Literacy Task Force, which included twenty-two professors and graduate students from several departments and colleges across campus, issued its Final Report and Recommendations. Contained therein are its definition and elaboration of quantitative literacy:

> Definition: Quantitative literacy is the ability to formulate, evaluate, and communicate conclusions and inferences from quantitative information.
> Elaboration: Quantitative literacy employs analytical arguments and reasoning built upon fundamental concepts and skills of mathematics, statistics, and computing. Quantitatively literate MSU students will be more empowered members of a global society through their ability to represent and critique their world (Estry and Ferrini-Mundy, 2005).

The definition was intentionally broad, instead of specific to any particular major, profession, or class of individuals. This definition was grounded in the knowledge and expertise of the diverse task force in addition to the relevant literature and similar definitions at other institutions. From this review of other programs and their definitions, it became clear there were several different ideas (quantitative reasoning, mathematical proficiency, mathematical literacy, statistical literacy, and numeracy) that each captured, in part, the goals set forth for quantitative literacy at MSU by the task force. Two central pieces of literature in the development of the definition were the Elements of Quantitative Literacy (Steen, 2001) and Strands of Mathematical Proficiency (National Research Council, 2001).

Quantitative literacy (QL) as a construct was dissected to help establish expectations, goals, and an initial definition to determine if the ideas embodied by the term were appropriate for MSU graduates. There were a number of colleges and universities that had Quantitative Reasoning programs and the task force wanted to be clear how the terms "quantitative" and "literacy" each played a part in developing this new program. Although these specific definitions were not highlighted, they represent the essence of the discussion.

As to the term "quantitative," QL is not exclusively a subset of arithmetic, mathematics, or statistics. "[It] is the ability to understand and use numbers and data in everyday life" (Madison, 2003, p. 153). Although the previous definition includes references to "numbers and data," QL is of course not synonymous with "statistics." Several suppositions of the task force support recognition of the quantitative aspect of QL as more than any one subject in mathematics or statistics (MSU Task Force, 2005) and not the result of a certain course(s) or set of skills that can be mastered (Hughes-Hallett, 2003). Following from this, QL is a frame of mind and set of skills that allows one to analyze and critique the world and make decisions using quantitative reasoning, placing the emphasis on how one uses the mathematics, statistics, and computing one knows, rather than on this knowledge itself.

Turning to the term "literacy," the task force definition can be interpreted as a synthesis of three types of literacy: conventional literacy, functional literacy, and critical literacy. Functional literacy, quite simply, is the ability to use reading, writing and arithmetic in everyday settings including, but not limited to, completing a job application, comparison shopping, and understanding a credit card application, including the annual percentage rate for interest, or the results for a political poll printed in a newspaper (Tozer, Violas, Senese, 1995). This view of literacy goes beyond the minimal "basic skills" of conventional literacy, but not enough for students to develop a questioning attitude and to participate fully in the democratic process. Critical literacy "means to approach knowledge critically and skeptically, see relationships between ideas, look for underlying explanations for phenomena, and question whose interests are served and who benefits" (Gutstein 2006, p. 5). For the MSU Task Force it was clear that the expectations must include rigor so QL was not mistaken for the basic computation skills of conventional literacy. As such there was an expectation that QL would be a value-added model developed over the entire college experience, with no assumption that high-school mathematics preparation alone was sufficient for the development of college-level QL.

The Current Project

Research Question

What is the quantitative literacy of our students?

That a large number of students place into developmental mathematics (see Estry and Ferrini-Mundy 2005) is often used as the evidence that students exhibit an insufficient level of QL. The assessment tests used to place students into mathematics courses, however, assess arithmetic and algebraic knowledge and ability, not quantitative literacy. In order to determine if the perceived lack of quantitative literacy is in fact real, we must use an assessment specifically designed to assess QL. Moreover, the fact that a student scores well on a mathematics placement assessment does not guarantee that the student is quantitatively literate. This project is the first step in diagnosing the need for, and focusing the implementation of, QL on MSU's campus.

Framework for Assessing Quantitative Literacy

In order to assess the QL level of our students, the current working group first had to create a framework for assessing QL. The group used the reports of the QL task force and the most recent MSU (CRUE) Council to Review Undergraduate Education (Bond and Phillips, 1988) to operationalize the definition of QL. The resulting framework has three dimensions: level, goal and domain. Each item on the assessment tool was classified along each of the three dimensions.

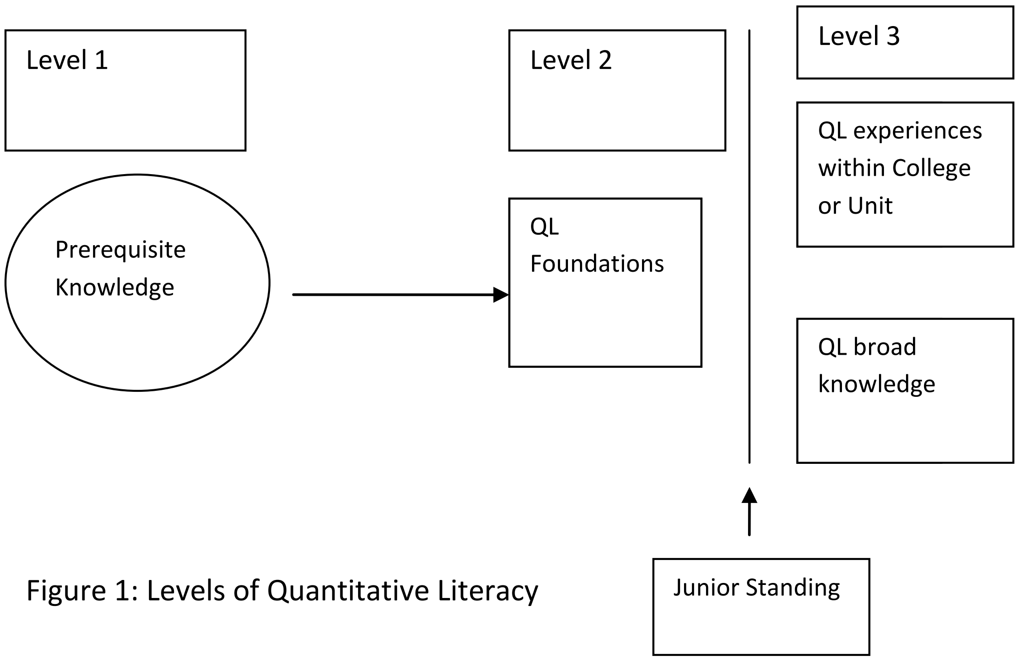

Figure 1: Levels of Quantitative Literacy

Levels of Quantitative Literacy

Our framework for assessing Quantitative Literacy development at MSU has three tiers, based on the notion that students should have the opportunities to develop and improve quantitative literacy across their entire college experience in multiple disciplines and contexts. The model for the levels was included in the QL Task Force report and is pictured above in Figure 1. The **prerequisite level** includes a fundamental understanding of mathematical, statistical, and computational concepts generally delegated to K-12 education or developmental courses at the college level. The **foundational level** expands this knowledge base to include outcomes typically from introductory computer science, mathematics, and statistics coursework. It was proposed that the prerequisite and foundational levels must be met prior to junior standing at the university so that these skills could be further utilized in upper-level courses. The **applied level** will involve the completion of a set of courses and/or projects which allow students to meaningfully apply their mathematical knowledge into an applied context, both general and major-specific.

Goals for Quantitative Literacy

Based on the reports of the QL Task Force and the CRUE, the current working group on the assessment of quantitative literacy has identified five big ideas that capture the goals for quantitative literacy at Michigan State University:

1. **Reading Graphical Displays:** Read and interpret representations of quantitative information such as tables, charts and graphs.
2. **Modeling Real World Phenomena:** Use multiple representations to model real world phenomena.
3. **Solving Real World Problems:** Solve real-world problems that require
    a. application of geometric properties
    b. application of algebraic properties
    c. application of probability and statistics
    d. use of computer software
    e. critical thinking skills
4. **Justifying Conclusions:** Explain and justify conclusions made with quantitative information including determining the appropriateness of the conclusion
5. **Critiquing Research Design:** Critique the appropriateness of a research design relative to the research objectives.

Domains of Quantitative Literacy

The domains of Quantitative Literacy reflect the content strands from PSSM (2000) and one of the dimensions of QL identified as *Skills needed for QL* by Steen (2001). Topics drawn from Arithmetic, Algebra, Geometry & Measurement, Data, Statistics, & Probability, and Computing form the subject matter knowledge essential for developing Quantitative Literacy. These domains represent an expansion from the traditional geometry, algebra, trigonometry, and pre-calculus/calculus (GATC) curriculum. GATC marginalizes many statistical and probabilistic topics that are used to analyze real-world data, instead focusing on a student's preparation for calculus and other **traditional** college mathematics courses (Steen, 2001).

<u>Instrument for Assessing Quantitative Literacy</u>

During the summer semester of 2007 an instrument was designed to assess the quantitative literacy of undergraduate students at MSU. Each item was classified on the basis of domain, goal and level. Those classifications are given in Table 1. In addition, care was taken to create items that represent authentic assessment. That is, the measurement of student performance is on tasks that are 1) relevant to life outside the classroom, 2) of interest, believable, and credible and 3) inclusive of topics important to quantitative literacy. This 16-item assessment that was developed contains questions from two levels and five domains written to measure the quantitative literacy. Only three of the five goals were examined in any depth.

TABLE 1: Classification of questions by domain, goal, and level of QL.

Domains	Arithmetic		Algebra		Geometry & Measurement		Data, Probability & Statistics		Computing	
Goals/Levels	Pre	Found	Pre	Found	Pre	Found	Pre	Found	Pre	Found
Read				1				3		
Model				1				1		1
Solve	2	1	1	1	2	1				
Justify								1		
Critique										

Below are two examples of assessment items. Both questions assess the goal of solving real world problems at the pre-requisite level. The Garment Question is from the domain of Arithmetic and the Tile Question is from the domain of Geometry & Measurement.

Garment Question- A garment is on sale at 40% off the original price. At checkout, the store will take an additional 20% off the sale price. Your discount from the original price is what percentage?

a) 60% b) 8% c)52%
d) Can not be calculated without knowing the original price of the garment.

Tile Question- Tom wants to replace the floor in his laundry room. His laundry room is a 3 ft by 6 ft rectangle. He is using square tiles that are 9 inches on a side. If one package of tiles contains 10 tiles, what is the minimum number of boxes he needs to buy?

a) 2 boxes b) 3 boxes c) 4 boxes d) 5 boxes

Participants and Protocol

In the fall of 2007 the assessment was given to more than 400 participants in two large-lecture sections of statistics courses at MSU. One course has no calculus pre- or co-requisite, while the second has a calculus prerequisite. For the purpose of this paper Group A (n=95) will be identified as No-calculus requirement while Group B (n=318) will be identified as Post-calculus. Students were given 30 minutes to complete the multiple-choice assessment during class time in the first week of classes.

Results

Due to some of the challenges we faced in executing this study, which will be discussed in more detail in the following section, we include only summary statistics of the data collected to date. Summary statistics for the assessment as a whole and for the two example problems given above are shown in Table 2. There was a difference of 0.55 points in the average scores of the two groups (SE = 0.30). This is especially interesting given that there were two items that specifically tested knowledge of rate that should advantage the calculus trained students. About one-half of the 0.55 difference is attributable to the better performance of Group B students on the two rate questions. Although one may assume the students having had calculus, or more advanced math, may have a general advantage, this was not realized on the Tile question.

TABLE 2
Average Number of Correct Questions and Percent of Students Correct for Sample Questions

	Mean	Std. Dev	Percent correct for Garment	Percent correct for Tile
Group A - No Calc	7.18	2.54	49.47%	58.95%
Group B - Calc	7.73	2.60	57.86%	53.77%

Discussion: Research and Assessment Challenges

This is a preliminary report of a pilot study of an instrument designed to assess QL. While the sample size is large, we have given an exploratory data analysis of the data because we are aware of certain issues with the data and data collection. These issues are: non-consent, motivation of subjects, and lack of background information of subjects.

In terms of non-consent, a number of individuals in each class did not consent for their data to be used in the study. For Group B, 3.93% (13 students) declined consent; 12.84% (14 students) of group A declined. Although we cannot comment on the scores of those students, it has been observed, in unpublished work that has been submitted to the MSU Institutional Review Board (IRB) for human subjects research, that those students who fail to consent for classroom data to be used in research tend to be weaker students in general than those who do provide consent. We, therefore, cannot know what effect removing the data associated with these students would have on the analysis.

A second issue with the data collected for this study is the motivation, or lack thereof, of students to do well on the assessment. Although this assessment was given in the classroom, there was no grade for performance attached. In Group B students were given 10 points for completing the assessment, regardless of the number of questions they answered correctly. Only half the class was asked to complete the assessment; the others were given another short assignment for which they were also awarded 10 points based on completion. Because students knew that they did not need to answer correctly to earn the associated points, it is difficult to know whether they were expending effort to attempt to answer the questions correctly. While the distribution of responses does not indicate that the students, as a group, were guessing randomly, it is possible that had there been more motivation to answer correctly, our results might have been different.

A final concern regarding the data is the lack of background information that we currently have for the participants. While Personal Identification Numbers (PIDs) were collected during the consent process so their results could be matched to their academic records at the university, at this time we have not yet received these data from the university. One of the records we hoped to have prior to this presentation was the coursework completed by the students in high school and at the university. It is possible that some students had previous coursework, for example, in statistics and probability. In this paper we have compared two groups of students based on the calculus requirement of the course. At this time it is unknown how many of the students in the course without a calculus prerequisite had actually taken calculus. In addition, we do not know the relative G.P.A.s of the students either overall or in courses in which they would have developed quantitative literacy skills. This information might explain the lack of differences we noted between the groups. Furthermore, more than 30% of our assessment contained questions from the Data, Statistics, and Probability domain of QL. Previous coursework in high school statistics (A.P. and non-A.P.) may have given some the advantage on these questions, raising their overall scores. This information might also contribute to an understanding of the results we obtained.

Directions for Future Research

In this paper we have discussed our process of developing a framework for assessing quantitative literacy at MSU. We began by operationalizing the task force definition, and then designed our assessment based on the goals and domains enumerated in the framework. After each pilot group was tested, we used the data to revise existing and create new items. The current assessment includes questions from several domains (Arithmetic, Algebra, Data, Statistics, & Probability, Geometry & Measurement, as well as Computing) at the prerequisite and foundational levels. We reported marginal differences by question and in overall performance by classes for which calculus was and was not a pre- or co-requisite. Statistical comparisons are not included due to some the concerns with the data that are noted in the previous section.

As we move forward with this research we will continue to write and revise assessment items and conduct additional testing in pre-service mathematics content courses, developmental mathematics, and non-mathematics intensive majors. We will analyze the reliability of the instrument alone and in combination with parallel instrument designed on campus to measure scientific reasoning. In addition, we plan to survey faculty members for their input on their vision for QL at MSU and to assess the validity of the instrument in measuring the construct of quantitative literacy. Finally, we will be conducting our first campus-wide testing of incoming students during Academic Orientation programs during the summer of 2008. Once we have collected those data and the academic background information on all of the subjects, we will begin analyzing the data for factors that indicate statistically significant differences in the level of quantitative literacy between student groups.

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