Pedagogical Challenges of Quantitative Literacy

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Abstract

This paper is based on two years of work in developing and delivering an ever-fresh, real-world based course that starts students down a path toward quantitative literacy (QL). Numerous pedagogical challenges have been encountered, but none more significant than the habits students have acquired from traditional courses in mathematics and statistics. Discussions of articles from current newspapers and magazines keep the course fresh but offer significant challenges for the instructor both in class and in assessment. The author has developed a list of characteristics that seem necessary for QL-friendly courses. These include: freshness, few formal algorithms, venues for continued practice, and emphasis on number sense.

Keywords: literacy, numeracy, pedagogy

1. Introduction

Quantitative literacy (QL) is a habit of mind, and, consequently, achieving QL requires both extensive interaction between students and teachers and practice beyond school. At the collegiate level, we are concerned with a high level of QL, befitting persons with baccalaureate degrees, analogous to Cremin's (1988) liberating literacy, as opposed to inert literacy. Therefore, the QL we seek includes command of both the enabling skills needed to search out quantitative information and power of mind necessary to critique it, reflect upon it, and apply it in making decisions.

Because students' educational experiences in quantitative matters have been dominated by courses in mathematics, and perhaps statistics, their inclination is to believe that instruction in QL should be similar to instruction in mathematics. Fortunately or unfortunately, in this author's experience, that is not the case, and some of the habits learned and attitudes formed in mathematics classes are actually obstacles to achieving the QL habit of mind. Furthermore, the teaching practices of traditional mathematics that engender student habits and beliefs seem to be ineffective in educating for QL. These practices range across the instructional spectrum, from learning goals and curricular materials, to conduct of classes and assessment of progress. The purpose of this essay is to detail some of the manifestations of the obstacles these attitudes, practices, and habits that the author has observed over the past two years while developing a QL-friendly mathematics and statistics course. These are not research results, but rather hypotheses based on observation, but nevertheless strongly held by the author.

2. The Challenges

1. QL is a habit of mind rather than a content-rich academic discipline.
2. High quality, effective curricular materials are scarce and scattered.
3. Students believe that QL is mathematics and behave as they do in traditional mathematics courses.
4. Abstracting generalities from contextual examples is difficult pedagogy.
5. Students expect template problems and homework exercises that match the template, and template problems are antithetical to QL.
6. Students believe QL is mathematics and therefore deem it not relevant to their lives and set apart from other areas of study.
7. Learning goals for QL are elusive.
8. Developmental levels of QL are neither understood nor articulated.
9. Assessment of QL requires authentic situations.
10. Performance standards for assessment are not established.
11. Multiple contexts challenge QL faculty and student understanding and knowledge.
12. Course material must be fresh and engaging.
13. Excursions into political and social issues are sometimes delicate and mysterious.
14. Mathematical and statistical concepts occur repeatedly and unpredictably.
15. Use of technology is essential but often foreign to students.
16. Mathematics and statistics encountered is usually elementary.
17. QL requires practice beyond school.

3. The QL-Friendly Course

Beginning in summer 2004 I developed a course based on mathematical and statistical reasoning required to analyze and criticize various newspaper and magazine articles. I call the course QL-friendly. I had been collecting the articles since 2001 when I began working with Robert Orrill and Lynn Steen in a national QL initiative by the National Center on

Education and the Disciplines. I first offered the course in fall 2004 as an experimental version of a traditional mathematics course called Finite Mathematics, which is designed primarily for business students. This first section had 26 students from various majors in the arts and humanities, and the course (as a section of Finite Mathematics) satisfied the mathematics requirement for the Bachelor of Arts degree in the Fulbright College of Arts and Sciences at the University of Arkansas. The second and third sections of this course, taught in spring 2005 and fall 2005, each had 40 students (the class maximum), almost all of them journalism majors. We focused the enrollment on journalism majors for several reasons: there were sufficiently many journalism majors to fill the class, the course met a requirement for their degrees, using news media as class resources gave the course a professional dimension for the students, journalists are somewhat noteworthy in their avoidance of quantitative analysis, and the journalism faculty were particularly supportive. However, this experience convinced me that students from a variety of disciplines provided a better audience for the class. Consequently, in spring 2006, we opened the course to all students, and 40 enrolled, mostly from arts and humanity majors but also several elementary education majors. By this time, the course had its own number and its own title, Mathematical Reasoning in a Quantitative World, and met the requirement of degrees in the arts and humanities. It could be called mathematical and statistical reasoning, because I include statistical reasoning as a part – a big part – of the reasoning in this course.

In each of the four iterations of the course, I have arranged the class loose leaf notebook (given to each student) into 9-11 lessons. Each lesson section contains brief explanations of mathematical concepts, 3-10 articles, and exercises on the concepts and articles. The lessons in the current version of the course are entitled:

- using numbers**;**
- percent and percent change;
- linear and exponential growth;
- indices and condensed measures;
- graphical interpretation and production**;**
- counting;
- probability, odds & risk;
- weights and geometrics measurement; and
- weather maps, measurement and indices.

There are several features of the course that I believe important. Three of these follow. First, I believe the class materials that provide the contexts for the mathematics and statistics problems must be authentic. Consequently, I wanted the newspaper and magazine articles not only to be authentic but to appear authentic. I first believed that this meant that they should be copies of real news print, but I soon learned that today's students are just as likely to view printed versions from Internet sources as authentic as they are to view news print thusly. One major reason for this is the students' experiences, but another is the fact that few of them read newspapers and magazines regularly if at all. However, even if the students are not intimately interested in a topic in a recent newspaper or magazine article, they hardly can deny that they should not understand it. Even though they may ignore the printed news media, they still accept it as a part of their world.

Second, I believe that the source articles must be fresh. The issues should be currently in the public discourse, and that discourse is rather fleeting, especially for college-age students. For example, the social security debate of a couple years ago was of limited interest to these students when it raged and is now passé. As this is written, the rising cost of gasoline and increasing interest rates are current. Consequently, the once-hot debate about increasing automobile gasoline efficiency for environmental reasons is now recast as a burning economic issue. Perhaps that will change by the fall term, prompted by Al Gore's movie, *An Inconvenient Truth*, pointing to the evidence of global warming. Keeping course material fresh offers a new challenge of producing textbook materials. Currently, my thinking is that the best I can do is to provide a skeletal framework for the course in terms of a textbook and require that the framework be filled out with fresh news materials.

Third, the students must be engaged in the material to a significantly larger extent than they are engaged in traditional mathematics or statistics courses. In order to do this, I offer students bonus credit for bringing to class articles that have interesting quantitative content. Students are asked to be able to comment on the articles they bring, and questions are fair game for comments. Sometime I take quiz questions from articles brought by students, so there is more reason to listen. We display the articles using an opaque projector so that all can see and read. This often challenges me as instructor to be able to clarify or answer questions extemporaneously, but I am willing to say I do not know, and I regularly read newspapers and have for many years. Often the students are more interested in the non-quantitative issues in the articles, and I feel obligated to allow some of that discussion. For example, two students brought the same article to class one day in this past spring. The article was a report on a faculty committee study of grade inflation. Aside from discussing the quantitative data and its representation in a graph, the students insisted on stating their own views of causes and what they considered fallacious reasoning by the faculty committee or the article writer. Allowing those discussions to proceed together is making an important connection between the quantitative issues and the non-quantitative issues, absolutely essential in my view to erasing the belief that mathematics is a world to itself.

3.1 Characteristics of a QL-friendly Course

My experience with the course I have developed over the past two years – which I call QL-friendly – has led me to a few conclusions about desirable characteristics of such courses, and, on the flip side, some conclusions about why traditional courses are not QL-friendly. Some of these characteristics are mentioned above, but I reiterate them here.

- Mathematics is encountered in many contexts such as political, economic, entertainment, health, historical, and scientific. Teachers will require broader knowledge of many of the contextual areas.
- Pedagogy is changed from presenting abstract (finished) mathematics and then applying the mathematics to developing or calling up the mathematics after looking at contextual problems first.
- Material is encountered as it is in the real world, unpredictably. Unless students have practice at dealing with quantitative material in this way they are unlikely to develop habits that allow them to understand and use the material. Productive disposition as described by Kilpatrick, Swafford and Findell (2001) is critical for the students.
- Much of the material should be fresh -- recent and relevant.
- Considerably less mathematics content is covered thoroughly.
- The mathematics used and learned is often elementary but the contexts and reasoning are sophisticated.
- Technology – at least graphing calculators – is used to explore, compute, and visualize.
- QL topics must be encountered across the curriculum in a coordinated fashion requiring those encountered in a QL-friendly course to make cross curricular connections.
- An interactive classroom is important. Students must engage the material and practice retrieval in multiple contexts.

The above characteristics translate into reasons why I believe that traditional mathematics and statistics courses are not QL-friendly. These reasons include:

- Emphases on components not processes
- Lack of mental constructs in lower level courses
- Lack of venues for continued practice beyond the course
- Not organized like the real world
- Tend to degenerate to methods and procedures
- Develop template problem expectations
- Not enough ambiguity
- Not enough interpretation and reflection

4. Student Expectations about Mathematics

Many students do not believe that mathematics has very much to do with their everyday lives. Part of this is due to the continued separation between the formal mathematics of school and the mathematics of everyday work, including the mathematics of commerce. As this author has written elsewhere (Madison, 2004), this separation has existed since there has been the two mathematics. Students hear and, on the surface, believe that the applications of mathematics are extensive and extraordinarily important and have been for centuries. So why do many lament that the mathematics of school and early college – geometry, algebra, trigonometry, and calculus (called GATC by this author (2004)) – has little relevance to their lives? There are several reasons. First, most college students never study mathematics to sufficient depth to see the applications of the GATC sequence to science and engineering. Second, traditional GATC mathematics courses allow few authentic applications to real world contexts. Third, most of the challenging mathematics of everyday life involves application of middle school mathematics (arithmetic, proportional reasoning, and measurement) in sophisticated contexts (e.g. health risks or economic rates of change).

Most higher education institutions include among the learning goals for students one or all of critical thinking, analytical thinking, and quantitative reasoning. Very often, both critical thinking and quantitative reasoning (or QL) will be listed as goals, reinforcing students' beliefs that these are disjoint constructs. Of course they are not (See (Bok, 2006), for example.), and students, who are usually highly receptive to critical thinking, would be more receptive to QL were they convinced it was a part of critical thinking. Students do not believe that quantitative thinking is critical thinking, and their experience in mathematics classes is strong evidence of that. To them, mathematics is not something for critiquing, nor is it subject to critiquing itself; mathematics is rigid, unforgiving, and absolute. There is truth in this, and to the extent it is true, that separates mathematical reasoning from quantitative reasoning.

Because students believe that mathematics is not relevant to understanding their world and is not a vehicle for teaching them how to reason, they often approach required college mathematics courses with at most a commitment to make a grade rather than understand and gain a step up the ladder of life readiness. This engenders the oft voiced attitude, "Just tell me how to work the problem and I will practice with homework exercises and learn it for the tests." At that point, effective education for QL is blocked. Why so? Well, problems for the everyday world do not come in groups of five that match some *template problem*. Instead, one rarely encounters the same problem situation problem multiple times in succession.

By and large, neither students nor faculty act as if they believe that mathematics has much to offer in terms of cross-cutting competencies such as critical thinking or communication. Reasoning is accepted by mathematicians as a critical feature of developing or using mathematics, and students believe that reasoning is an important process for them to master. However, in my experience, students see mathematical reasoning as distinct from reasoning in other domains, another manifestation of the separation of mathematics from the rest of the world of many students. To many students, mathematics is a subject all on its own, and faculty are not much different. Although mathematics faculty recognize the incredible array of uses of mathematics in the real world, most of these uses are in contexts well out of reach of beginning college students.

One of the aspects of mathematics that tends to cause students to treat it as apart from the rest of their world is the logical structure of mathematics and the way of knowing in mathematics. This way of knowing, through formal proofs, creates rigidity and formalism somewhat alien to many students thinking. One aspect of that is the way definitions of terms/concepts occur in mathematics as opposed to the way definitions occur from student experience in other realms. Many terms in mathematics – e.g. rational, proportion, complex, derivative, integral, similar, ring, field, closed, open, etc. – have meanings derived from student experiences that are not only different in content but in specificity from the stipulated definitions in mathematics. These two types of definitions as discussed in the literature, for example, (Edwards & Ward, 2004) – extracted and stipulated – sometimes get confused, but that is not the biggest impact in my opinion. This difference reinforces students' inclination to believe that mathematics is not part of the real world because even the words have different and mysterious meanings. The variability of extracted meanings of a word and confusion of those meanings with the stipulated meaning in mathematics gives both conceptual and psychological barriers to understanding and using mathematics.

4.1 Is My Course a Mathematics Course?

The answer in my view is fundamentally irrelevant except that students believe the course is a mathematics course and behave accordingly. Numerous students have said to me, "This course is not like any mathematics course I have had before." Traditionally, mathematics courses, especially those in the GATC sequence, have specific mathematical content that leaves little room for authentic applications much less any forays outside to other disciplinary or cross-disciplinary topics. This separates college mathematics courses from those in most other disciplines, where excursions into multidisciplinary topics are commonplace. Courses that are QL-friendly, in my view, must cross disciplines and venture frequently into popular culture. This, of course, may make the course more difficult to teach for faculty accustomed to traditional mathematics courses.

5. Teaching/Learning/Assessing a Process

Achievement of QL requires a level of mathematical proficiency similar to that described in the National Research Council's *Adding It Up* (Kilpatrick, Swafford, & Findell, 2001, p. 5). Although focused on K-8 mathematics, the *Adding It Up* characterization of mathematics proficiency as having five strands can help to understand QL. The five strands – conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition – probably have different emphases in QL than in mathematics, but one can clearly see the need for each in solving a canonical QL problem or understanding a QL situation. In my experience with students, one of these five may be more critical for QL than the others, and that one is productive disposition. As described in *Adding It Up* (p. 5), productive disposition is the habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy. This strand of mathematical proficiency, probably more than the others, is weakest in the students I have taught. As outlined above, they do not see mathematics as useful, and they have very weak beliefs in their efficacy in mathematics.

A canonical QL situation involves several steps, some of which are encountered in traditional mathematics or statistics courses, but rarely is the process with all the steps part of these courses. The steps can be described as follows, where I have indicated a critical strand in mathematical proficiency that seems necessary.

- Encountering a challenging contextual circumstance, e.g. reading a newspaper article that contains the use of quantitative information or arguments. (Productive disposition)
- Interpreting the circumstance, making estimates as necessary to decide what investigation or study is merited. (Adaptive reasoning)
- Gleaning out critical information and supplying reasonable data for data not given. (Productive disposition and conceptual understanding)
- Modeling the information in some way and performing mathematical or statistical analyses and operations. (Strategic competence and procedural fluency)
- Reflecting the results back into the original circumstance. (Adaptive reasoning)

These steps often require careful reading of continuous prose and graphical representations or other discontinuous prose, using mathematics or statistics, and then interpreting and critiquing the original prose in light of the mathematical results. Critical reasoning (closely akin to adaptive reasoning) is required throughout. Students are not expecting this complicated process because their previous mathematics experiences have been narrower and better defined. Consequently, one struggles with breaking the process into bits and pieces and teaching these separately. Frequently, the third phase gets the most attention because it is the process of traditional mathematics and statistics courses.

5.1 Assessment Difficulties

Setting learning goals and achievable objectives for QL is a major challenge. The big overarching goal is easy to state but very difficult to break down into achievable objectives. It is easy to say that QL is the ability to understand and use quantitative measures and inferences that allow one to function as a responsible citizen, productive worker, and discerning consumer. It is quite another to break that down into development steps and achievable learning objectives that achieve the desired understanding and skills.

Assessment items must be authentic, and according to Grant Wiggins (2003) that requires that they be complex, realistic, meaningful, and creative, and have value beyond school. One can use assessment items that are narrower, say focusing on the basic mathematics or statistics skills and knowledge needed for QL. If one knows what these skills are, then assessing them is only a piece of the bigger assessment task. As Grant Wiggins has pointed out, assessing QL is analogous to assessing whether a person is a good soccer player. One can assess individual skills required in soccer, but the proof comes with actually playing the game.

Even after deciding on authentic assessment items or processes, two challenges remain. What will be valued in scoring? Are reading, interpreting, computing, reflecting, and writing all parts of what will be evaluated? They are all parts of QL, and the challenge of scoring all is substantial. The second challenge is determining levels, or standards, for proficiency in QL. Since QL is society dependent and certainly changes over time and place, the proficiency standards of the past or of other societies are not necessarily appropriate. Few people will be able to successfully handle quantitative issues across all of the possible domains in US society. Consequently, one has to decide on what domains are common enough to be included in setting standards. Clearly, the challenges are quite daunting.

6. Conclusion

The foregoing could be discouraging, but one should remember that QL presents a new educational challenge, similar to those faced with reading, writing, and more general critical thinking. The background for QL, firmly attached to mathematics, seems to make the problems more difficult. The circumstances that demand higher levels of QL are largely the result of developments stemming from higher education. Consequently, higher education has an obligation to address the problem, and it seems that this will require new cross disciplinary efforts. Mathematics is a reluctant promoter of QL education, most likely because it is so different and some see QL as a threat to mathematics. College and university mathematics will need to change pedagogical approaches if it is to successfully lead QL education. If it does not, then I believe QL is a major threat to collegiate mathematics because QL-friendly courses will likely replace courses such as college algebra that are now being used ineffectively as general education courses, and a major fraction of collegiate mathematics enrollments are in college algebra and other algebra courses.

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