Some Historical Notes

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Introduction

The classic children's book "The Little Engine that Could" provides an interesting framework to use in reflecting on the movements in the past which have led up to the quantitative literacy programs at colleges and universities today. Whether there was a "happy" train to begin with which was trying to haul "toys and food to the other side of the mountain" is an unanswered question, but certainly there were some "well-intentioned" trains which had engines that either stopped "with a jerk" or slowed to a crawl while yearning for the aid of other engines.

In looking at quantitative literacy through the years, this essay seeks to familiarize the reader with some of the books and proceedings related to quantitative literacy (past and present), define some terms regularly used in discussing quantitative literacy, trace the development of the content and pedagogy for quantitative literacy programs as it evolved through the work of the Mathematical Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM), note connections of the quantitative literacy "movement" with other curricular movements being advocated or occurring in the mathematical community, and link these movements with changes in text material and pedagogical sources.

Early Movements by the MAA

For the past fifty years the natural MAA curricular group to consider mathematics for general education in college has been its Committee on the Undergraduate Program in Mathematics (CUPM—first called just CUP). In the summer of 1954, even before Sputnik, the committee sponsored a writing group for the development of an experimental general mathematics text for all "normally" prepared first year college students. The text *Universal Mathematics* appeared in two parts with writing continuing from 1954 to 1958. The work presupposed students having at least two, and preferably two and one-half units of high school mathematics (a rather common attainment for college-bound students in the 1950s). It seems that the intent, even by its title, was some minimal expectation for all college students, but the term quantitative literacy was not used.

While there was some pilot-testing of *Universal Mathematics*, CUPM's attention was diverted to other matters in the years that followed. Its efforts led to the booklet *A General Curriculum in Mathematics for Colleges* (GCMC), aimed at describing "a basic mathematics program … to accommodate today's diversity of students and their objectives." In connection with a section in that report called "One-year mathematics to satisfy a B.A. requirement" remarks are made that say no description is given of a special year-course in mathematics appreciation for students at liberal arts colleges, because the Committee felt it better for such students to pursue either a year of calculus or a semester of calculus followed by a course in probability which used the calculus. The report section then provides a few remarks pertaining to college students in general, but makes the disclaimer that these "remarks do NOT have the force of a recommendation, since CUPM has not yet considered in detail this important curricular problem" (page 25). Further, the report is punctuated with concerns for the huge increase in the number of students going to college in the 1960s and the shortage of faculty available to cover the students' needs in mathematics.

In 1969 CUPM presented *A Transfer Curriculum in Mathematics* and advocated that Math A described therein have as an objective the development of mathematical literacy. Such literacy was defined as "the ability to read and understand mathematical statements and the ability to translate into mathematical language (making proper use of logical connectives) statements and problems expressed in ordinary English. Continual practice should be given to solving 'word problems' and in analyzing mathematical statements, with particular emphasis on developing the ability to understand and to use deductive reasoning." Math A was intended for those who needed reinforcement of their high school mathematics as well as preparation for calculus. It was not prescribed for all college students, nor was any specific course with a literacy objective set out for all college students. Math A was essentially a slower-paced version of the "old-fashioned" algebra, trigonometry, and analytic geometry. The 1969 report "deferred the consideration of lower-level or non-university parallel courses as a matter for further study." In January 1970, CUPM formed a Panel on Basic Mathematics to provide the "further study". The considerations of the Panel were presented in January 1971 in the booklet *A Course in Basic Mathematics for Colleges*. The thrust of the January 1971 report was to describe a single flexible one-year course, known as Math E, together with a linked mathematics laboratory.

Math E was proposed to replace some of the courses being taught below college algebra. The course's main aim was stated to be "to provide the students with enough mathematical literacy for adequate participation in the daily life of our present society." The associated laboratory was intended to be used to remedy deficiencies in arithmetic and provide opportunity for drill in algebraic manipulation. A secondary aim of the course was to provide enough algebraic skill and use of mathematical language so as to enable students who desired to do so to continue on to Math A. The course was expected to reach out to students of a higher age level and greater maturity than those coming immediately out of high school, ones for whom it might be their terminal course in mathematics. Still, the course was not viewed as defining literacy for all college students.

Continued efforts by CUPM on the curriculum for four years of college mathematics led to the publication in 1972 of *Commentary on A General Curriculum in Mathematics for Colleges*. The intent of the Commentary was to note that many of the 1965 suggestions were still relevant, but to also modify some 1965 suggestions because of recent developments. The 1965 statement about a special one-year course in mathematics appreciation for students in liberal arts colleges was repeated with the further view expressed "that mathematics is best appreciated through a serious effort to acquire some of its content and methodology and to examine some of its applications"(page 43).

Turmoil in the nation in the early 1970s led to many colleges and universities in America abandoning general education programs, "core" curricula, and even minimal competencies. The climate certainly was not ripe for CUPM to be making recommendations regarding mathematics for ALL college students. However, by 1978 there was revived interest in general education programs and minimal requirements for students, and CUPM formed a committee to consider the mathematical needs of ALL college students.

Connections with Influential Forces in School Mathematics

Concern for school mathematics and its teaching became a national concern when Sputnik streaked across the sky in 1957. The call went out for improved teaching of mathematics and an improved curriculum. CUPM had an active panel on teacher training for the schools, and a number of professional organizations went together to obtain National Science Foundation support for the School Mathematics Study Group (SMSG). In the summers of 1960 and 1961 SMSG writers produced materials for an improved K–12 curriculum. These were field-tested (many at university lab schools) and a longitudinal assessment was carried out in the period 1966 until 1970.

The SMSG materials were part of what came to be called "the new mathematics"—a movement at its best aimed primarily at teaching for greater understanding of the mathematical concepts at the school level where too often total rote had taken over as a teaching methodology. Unfortunately, the new materials were thrust by well meaning educational administrators on teachers who were not prepared to properly use them. The consequences of such actions led to a "back to the basics" counter movement seeking to reinstall the once prevalent rote memorization teaching methodology and intending to remedy the concern the public held because of problems evidenced largely on test results. These happenings came in the early 1970s when colleges and universities in America were abandoning general education programs and minimal competencies.

As the 1970s rolled on, the thrust of the two movements in school mathematics which appeared to be counter to each other was to be taken up by the leadership of the National Council of Teachers of Mathematics (NCTM) and its affiliated groups to articulate realistic and responsible directions mathematics programs should take in the 1980s. Using a series of studies funded by the National Science Foundation, two mathematics assessments of the NAEP, and an extensive survey of diverse sectors of the society (a project called PRISM), the NCTM made its recommendations in April, 1980 labeled *An Agenda for Action*. The Agenda consisted of eight recommendations--three of which were: 1) Problem solving must be the focus of school mathematics in the 1980s; 2) The concept of basic skills in mathematics must encompass more than computational facility; and 3) Public support for mathematics instruction must be raised to a level commensurate with the importance of mathematical understanding to individuals and society. Accordingly, the early 1980s yearbooks of the NCTM provided material supportive of the Agenda's implementation.

Actions of the Early Eighties

The report of the CUPM panel formed in January, 1978, was published in the American Mathematical Monthly in 1982. It noted that "In a relatively severe, but all too common form, ignorance of mathematics amounts to a form of 'functional illiteracy'." It sought to answer the question: What mathematics should every graduate of an American college or university know?

In response to surveys to determine the current status of minimal mathematics expectations at American colleges and universities, the panel found such great diversity that it could not describe an everywhere attainable goal. Its primary recommendation was: All college graduates, with rare exceptions, should be expected to have demonstrated reasonable proficiency in the mathematical sciences.

To implement this recommendation the panel stated that every college or university should formulate, with adequate concreteness, what this "reasonable proficiency" should mean for its students; define how students should demonstrate this proficiency; and establish this demonstration as a degree requirement. It noted that: "The idea that all college graduates should be expected to have acquired a certain familiarity with mathematics rests in part on the well-founded belief that such a familiarity is necessary for effective functioning in contemporary life, and certainly for life in those spheres college graduates are most likely to enter." The ignorance the panel deplored, and the belief it expressed for students being able to function mathematically, were soon to receive national attention.

Both the MAA and the NCTM were working on reform in the teaching and learning of mathematics when *A Nation at Risk* was published by the National Commission on Excellence in Education in 1983. The publication of *A Nation at Risk* led to a flurry of activity including a retreat sponsored by the Conference Board for Mathematics at which the group endorsed the establishment of a set of standards to be drawn up by the NCTM and the formation of the Mathematics Science Education Board, an umbrella organization with representatives from the mathematical sciences, education, and industry and business that could provide overall direction for a broad reform movement in mathematics education.

Calls for Change and Reform

One by one reports issued at the national level pointed out that mathematically illiterate individuals would not be able to participate fully in the life of our contemporary society. Among such reports were: 1) *The Mathematics Report Card: Are We Measuring Up*? (Educational Testing Service, 1988); 2) *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (National Academy Press, 1989); 3) *50 Hours: A Core Curriculum for College Students* (National Endowment for the Humanities, 1989); and 4) *Moving Beyond Myths: Revitalizing Undergraduate Mathematics* (National Academy Press, 1991).

In March of 1989 while some of these reports were at the printers or still in preparation, the NCTM released its *Curriculum and Evaluation Standards for School Mathematics* (usually shortened to *The Standards*). These were to be followed in 1991 by NCTM's *Professional Standards for Teaching Mathematics*. Also appearing in 1991 was a report from the MAA Committee on the Mathematical Education of Teachers named *A Call for Change: Recommendations for the Mathematical Preparation of Teachers of Mathematics*.

Begun in 1980 the Educational Equality Project of the College Board aimed to strengthen the academic quality of secondary education and to ensure equality of opportunity for post-secondary education for all students. In 1983 (the year *A Nation at Risk* appeared) the Project published its *Academic Preparation for College: What Students Need To Know And Be Able To Do*. The basic competency in mathematics and the specific preparation in mathematics for a college entrant detailed in the booklet defined a level of knowledge and skills expected for all high school graduates intending to do college work.

Also in the early 1980s, the American Statistical Society joined forces with the NCTM to provide curriculum materials and in-service training so that mathematics teachers in the secondary schools could effectively incorporate basic concepts of statistics and probability into their teaching. Published in 1987, the resources carried the (unfortunate) title *Quantitative Literacy Series*. Their usefulness formed some background for the heavy data analysis perspective prevalent in strands of the NCTM *Standards*.

Another call for change in the 1980s was one to reform the teaching of calculus. After a remarkable session of discussion at the national professional meetings, the Sloan Foundation sponsored a conference/workshop at Tulane University in January of 1986 to develop alternative curriculum and teaching methods for calculus at the college level. The report on that conference *Toward A Lean and Lively Calculus* appeared in the MAA Note series late in 1986. In 1987 the Sloan Foundation funded a convocation *Calculus For a New Century* at the National Academy of Sciences, and the National Science Foundation supported the work of many calculus reform projects from 1988 through 1994.

Two other movements were prevalent in the 1980s which also impacted quantitative literacy issues. On the one hand there were those developing new liberal arts courses--many funded by the Sloan Foundation and aimed for elective use by students at four-year somewhat selective institutions, and on the other hand, there were those seeking to foster a new view of mathematics by the public. In the latter area, John Paulos published the book *Innumeracy*, whereas the television series funded by the Annenberg Foundation and the 1988 book *For All Practical Purposes* (now in Second Edition) simultaneously contributed to both movements.

Late in 1989 with *The Standards* now published by the NCTM, CUPM formed a committee on quantitative literacy requirements (hereafter referred to as the 1989 QL Committee) to frame recommendations which would mesh with the new pre-college standards and be realistically achievable in the college years. The Committee studied the many documents and books noted in this essay (and more), gathered input by conducting a series of well-attended sessions at the national mathematics professional meetings (including a debate on the use of college algebra as a requirement which was attended by about 200 people), gathered input and fostered debate through essays and announcements in national professional newsletter publications, considered the views of societies such as the Sigma Xi, sponsored a Focus Group discussion session reported in the 1992 publication *Heeding the Call for Change*, and studied writing on overcoming math anxiety, studied research available from mathematics educators—especially on problem solving—and conducted a massive poll of colleges and universities across the United States to determine the then current objectives and requirements concerning mathematics for all students.

The 1989 QL Committee's report which CUPM adopted in January 1995, was NOT a distillation of the current status in the mathematical community regarding quantitative literacy, but a visionary document with recommendations supported by suggestions and approaches intended to enable the recommendations to be ACHIEVABLE. In some ways the current volume is a testimony to their usefulness.

QL Terms Defined

The 1989 QL Committee agreed that every college graduate should be able to apply simple mathematical methods to the solution of real-world problems. It further defined a quantitatively literate college graduate in terms of five capabilities. That is, such a graduate should be able to:

1) Interpret mathematical models such as formulas, graphs, tables, and schematics, and draw inferences from them.
2) Represent mathematical information symbolically, visually, numerically, and verbally.
3) Use arithmetical, algebraic, geometric and statistical methods to solve problems.
4) Estimate and check answers to mathematical problems in order to determine reasonableness, identify alternatives, and select optimal results.
5) Recognize that mathematical and statistical methods have limits.

While these capabilities could be attained at varying levels, the 1989 QL Committee further explained its intent and how these capabilities would mesh with *The Standards* of the NCTM for the college graduate. It noted that the college graduate should be expected to have deeper and broader experiences than those who only graduate from high school and that the level of sophistication and maturity of thinking expected should extend to reasoning which is commensurate with the college experience. "College students should be expected to go beyond routine problem solving to handle problem situations of greater complexity and diversity, and to connect ideas and procedures more readily with other topics both within and outside mathematics." Quantitative literacy for college students is seen in one sense as extending the concept of "mathematical power" in *The Standards*, including some mathematical content, but especially involving the ability to *use* concepts, procedures, and intellectual processes with a greater degree of versatility in approaching and solving problems or understanding quantitative information.

The British use the word "numeracy" to mean mathematical literacy, but in the American mind this word conjures up the notion of just number sense at its lowest levels and not the broader implications set by the definition above.

The 1989 QL Committee recognized that the introduction of a single course would not enable a college student to move from the high school attainment of mathematical power to the college level of quantitative literacy. Research by mathematics educators noted that there were five aspects of intellectual competency which provided a framework for problem solving. These included knowledge of concepts, fact, procedures, and strategies or heuristics, but also knowledge of how and when to use these in a manner which is effective and efficient. Further, students require practice in the acquisition of the habits of sense-making, interpretation, and exercise of control. Advocated in the 1989 QL Committee report was thus a PROGRAM in quantitative literacy. A quantitative literacy PROGRAM normally has the following components:

1) Explicit requirements of quantitative experience for college entry or for entry into courses or experiences which can be credited towards the baccalaureate degree;
2) Placement testing intended to help determine appropriate entry into the quantitative literacy program;
3) Foundation experience(s) to be accomplished ordinarily within the first year of the student's college work;
4) Further quantitative experiences in diverse contexts to be accomplished during a student's sophomore, junior, and senior college years so as to be interspersed throughout the work of these years.

The foundation experiences have generally been specific courses designated by colleges. They are intended to provide some resources in mathematics, but, more importantly, the knowledge of strategies and heuristics and guidance on when to use these in conjunction with the mathematical knowledge. FOUNDATIONS courses are intended to introduce students to the complexity of thought and maturity of thinking expected in the college experience; such courses should provide a background on which a student can draw as the student encounters mathematical thought and quantitative reasoning throughout the undergraduate curriculum. A quantitative literacy PROGRAM also includes CONTIN-UATION experiences. CONTINUATION experiences may be courses or projects or other aspects of the undergraduate curriculum which enable the student to PRACTICE the patterns of thought introduced in the FOUNDATIONS experiences. CONTINUA-TION experiences are an important component for enabling the student to genuinely extend the concept of mathematical power, and to more fully acquire the capabilities intended for the college graduate. Ideally, quantitative reasoning should permeate the undergraduate curriculum the same way quantitative data permeates society.

Content and Pedagogy

What should be the content of a "foundations course" or the nature of the so-called foundations experience? Whatever the actual mathematics involved in the foundations work, the content would be a solid introduction to the type of quantitative reasoning expected of the student. Thus the content involves the student actually DOING quantitative reasoning and not just being exposed to it. Depending on the accomplishment in mathematics of students as they enter college, the foundations work provides resources in mathematical knowledge, problem-solving strategies, interpretive forms, formulation of problems, and engagement in the use of these in genuine applications. As noted in the 1989 report, "there are plenty of mathematical topics with both utility and beauty, so beauty need not be sacrificed." The mathematical topics chosen should link with the prerequisite knowledge assumed. The content chosen also provides a basis for instructors in other fields to show students how to apply the foundation knowledge in gaining understanding in that field through quantitative reasoning. Certainly a FOUNDATIONS course cannot be the "old-fashioned" intermediate algebra or college algebra course or the "old-fashioned" liberal arts mathematics appreciation course.

The pedagogical approach for a quantitative literacy foundations course or experience is the study of mathematics in context. And the context is expected to relate to student interest, daily life, and possible future work settings even when the character is remedial. Of course, the expectation for a college student is to extend study to the completion of work beyond that of remedial character.

Teaching methods for quantitative literacy courses are not lecture and listen, but they may involve group work, projects, writing, and many of the approaches advocated by those in the calculus reform movement. In particular, teaching methods generally enable students to be actively involved in building their understanding while connecting with their prior ideas.

Historically, quantitative literacy courses have not been prevalent in colleges and universities across the United States. Instead there have been many remedial and many algebra courses taught which often aimed only at computational skills, and usually by means of rote learning. Or there have been liberal arts courses required which often covered the topics faculty WANTED to teach and/or provided a survey of mathematical ideas while doing little to develop the student's reasoning powers. In fact, they provided a hurdle to jump for degree requirements, but little "carry away" value.

Some selective four-year institutions seemed to be content with the mathematical knowledge their students had upon entry--namely four years of high school mathematics. These institutions appeared to be unconcerned with whether their students could use the mathematics they had studied in everyday quantitative settings or in their study of other disciplines in their college careers. The development of quantitative reasoning commensurate with the development of the college mind was not an agenda item for such colleges.

At the time the 1989 QL Committee gave its report there were very few foundations courses in the United States and certainly a real lack of text material or other course material available to support the introduction of such courses. Indeed, because of the varying levels of entry into foundations courses brought on by the varying level of mathematics required for admissions at colleges and universities, a wide variety of material was required. Since 1996 however, a number of colleges have introduced foundations courses for students whose college programs require no hierarchy of mathematics, and some text materials are now in second or third edition. Further, at a number of colleges, the liberal arts course, traditionally focused on aesthetic topics, has taken on a shift towards having at least a quantitative literacy component, and some "old" liberal arts mathematics books in latest editions have shifted to having such a component too. In fact, for some lower division courses commonly taught which might be entry points for students in a multiple entry quantitative literacy program, "reformed" text material provides a quantitative literacy component.

Whatever the entry point into a foundations experience, the 1989 QL Committee noted that the experience by definition should be a natural transition from the processes now listed as the new *Standards* towards the depth expected in a college career. These processes of problem solving, reasoning, connections, communication, and representation are to be introduced with appropriate mathematics in context in the foundations experience encountered early in college study and developed in continuation experiences as the college years unfold.

Conclusion

So where IS "the little engine that could"? Since the adoption by CUPM in January 1995 (and the publication in 1996) of the 1989 QL Committee report, a large number of colleges and universities have at least started on a quantitative literacy program by developing a foundations course. In the State of Illinois the adoption by the so-called Illinois Articulation Initiative of the viewpoint that a foundations course be included in its general education program has led many institutions to develop such a course. With foundations courses firmly in place, work can be done to foster continuation experiences.

Nationally, various efforts supporting quantitative literacy, noted in another essay in this volume, have kept the little engine moving a little faster. While some national reform efforts in mathematics at the collegiate level may be playing the role of having passed by a slow moving train, the little engine IS moving.

Among other movements the little engine is being helped by the work of the new NCTM *Standards*, and SIGMAA QL, the new special interest group formed at the January 2004 national MAA meeting should be greasing the wheels even more.

This volume itself should be helpful to colleges and universities in developing a quantitative literacy program more fully. In doing so, it will enable the little switch engine to haul "toys and food to the other side of the mountain" for many more college students!

References

Summer Writing Group of the Dept. of Mathematics, Univ. of Kansas, Universal Mathematics. (1954). *Part I: Functions and Limits*. Univ. of Kansas, Student Union Book Store.

Bennett, J. & Briggs, W. (2002). *Using and Understanding Mathematics: A Quantitative Reasoning Approach*, (2nd ed.). Addison-Wesley.

Cheney, L. V. (1989). *50 Hours: A Core Curriculum for College Students*. National Endowment for the Humanities.

College Board, Academic Preparation For College: (1983). *What Students Need to Know and Be Able to Do*. The College Board.

CUPM, (1971). *A Course in Basic Mathematics for Colleges*, MAA.

——, (1965). *A General Curriculum in Mathematics for Colleges*. MAA.

——, (1969). *A Transfer Curriculum in Mathematics*. MAA.

——, (1972). *Commentary on "A General Curriculum in Mathematics for Colleges"*. MAA.

——, (1982). "Minimal Mathematical Competencies for College Graduates." *Amer. Math. Monthly* 89, 266–272; reprinted in Steen, L., (1989). *Reshaping College Mathematics* (MAA Notes Number 13, pp. 103–108). MAA.

Davis, R.L. , ed. (1958 CUPM). *Universal Mathematics, Part II: Elementary Mathematics of Sets with Applications*.

Douglas, R. G., ed., (1986). *Toward a Lean and Lively Calculus* (MAA Notes Series #6). MAA.

Educational Testing Service (1988). *The Mathematics Report Card: Are We Measuring Up?* ETS.

Garfunkel, S.. ed., (1991). *For All Practical Purposes*, (2nd ed.). W. H. Freeman.

Goldberg, S., ed., (1990). *The New Liberal Arts Program: A 1990 Report*, Alfred P. Sloan Foundation.

Harrison, A. (1990). *Entry-Level Undergraduate Courses in Sciences, Mathematics, and Engineering: An Investment in Human Resources,* The Scientific Research Society.

Leitzel, J., ed., (1991). *A Call for Change: Recommendations for the Mathematical Preparation of Teachers of Mathematics*, MAA.

National Commission on Excellence in Education, A Nation At Risk (1983). *The Imperative for Educational Reform*, U.S. Government Printing Office.

NCTM, (1980). *An Agenda for Action: Recommendations for School Mathematics of the 1980s*, NCTM.

——, (1989). *Curriculum and Evaluation Standards for School Mathematics*, NCTM.

——, (2000). *Principles and Standards for School Mathematic*s, NCTM.

——, (1981). *Priorities in School Mathematics*, NCTM.

——, (1991). *Professional Standards for Teaching Mathematics*, NCTM.

NRC, (1989). *Everybody Counts: A Report to the Nation on the Future of Mathematics Education*, National Academy Press.

——, (1991). *Moving Beyond Myths: Revitalizing Undergrad-uate Mathematics*, National Academy Press.

Paulos, J. (1988). *Innumeracy: Mathematical Illiteracy and Its Consequences*, Hill and Wang.

Piper, W. (1998 reteller). *The Little Engine That Could,* Platt and Munk. (original entitled "I Think I Can" published in 1930).

Quantitative Literacy Series (1987). (A project of the American Statistical Association) Dale Seymour Publications.

Schufelt, G., ed., (1983). The Agenda in Action, *1983 Yearbook of the NCTM*, NCTM.

Sons, L. R., ed., (1996). *Quantitative Reasoning for College Students: A Complement to the Standards*, MAA.

——, (1992). "Reaching for Quantitative Literacy" in L. Steen, ed., *Heeding the Call for Change* (MAA Notes Number 22), MAA, pp. 95–118.

——, (October 1990). "What Mathematics Should Every College Graduate Know?" *UME Trends*.

Sons, L. R., Nicholls, P. J., & Stephen, J.B., (2003). *Mathematical Thinking and Quantitative Reasoning*, (3rd ed.). Kendall/Hunt.

Steen, L. A., ed., (1987). *Calculus for a New Century* (MAA Notes Series #8), MAA.