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**Quantitative Literacy:
Core Concepts**

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*Creating and Strengthening
Interdisciplinary Programs in QL*
MAA PREP Workshop
Macalester College
14 June 2005

Good evening.

Welcome to the MAA Prep Workshop: “Creating & Strengthening Interdisciplinary programs in Quantitative Literacy”

Our first order of business is to thank our host.

David would you please stand. David organized this workshop.

He organized the funding that made it possible for many of us to attend.

He is the chair of the MAA Committee on Undergraduate Programs in Mathematics (CUPM).

Please join me in thanking our host: David **Bressoud**.

Bernie would you please come up.

Please join with me in welcoming the President of the National Numeracy Network, **Bernie Madison**.

I want to say “Thank you” to the W. M. Keck Foundation for their grant to Augsburg College.

This grant was to “develop Statistical Literacy as an Interdisciplinary curriculum in the liberal arts.”

This grant has been used to create and field test new teaching materials and new tools.

The title of my talk is “**Quantitative Literacy: Core Concepts**”

I hope these concepts will be useful in designing and teaching QL courses and programs.

> Let’s start with some background >



The need for quantitative literacy has been well articulated by Lynn Steen, Bernie Madison, Linda Sons and many others.

These are three recent publications that articulate the need.

Everyone interested in QL should study these publications.

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**To grow, QL must be
Understandable & Teachable**

"Because of their education and training, most teachers are not prepared for or comfortable with the mathematics required for quantitative literacy."

"According to Johnny Lott, former president of NCTM, it is simply unrealistic to expect that teachers of other subjects will either know or understand what might be considered quantitative literacy."

"QL advocates need to be very clear about what all students need to know and be able to do...."

Quantitative literacy has an apparent weakness.

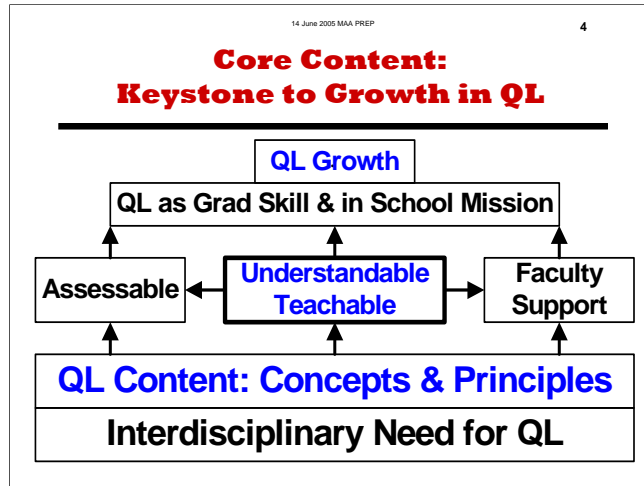
Q/L is hard to describe and very hard to define.

These quote are taken from publications on QL edited by Lynn Steen.

"Because of their education and training, most teachers are not prepared for or comfortable with the mathematics required for quantitative literacy."

"According to Johnny Lott, former president of NCTM, it is simply unrealistic to expect that teachers of other subjects will either know or understand what might be considered quantitative literacy."

"QL advocates need to be very clear about what all students need to know and be able to do..." Janice Summers.



We have the top and bottom of this pyramid.

The Top: We know where we are headed. We are clear on our goal.

Our common goal is to see quantitative literacy grow.

The bottom: We have the foundation.

The need for quantitative literacy has been documented.

The books by Steen and Madison have documented this very clearly.

So, we have the foundation and we know where we are headed.

 To support an interdisciplinary area, college-wide support is necessary.

To obtain college-wide support, the subject must have wide faculty support.

And in today's climate, it must be assessable.

Unless QL is a clearly-stated campus-wide goal,
 QL programs are likely to be transient.

Interdisciplinary programs cannot survive in a disciplinary world without strong college support

None of this will not happen unless quantitative literacy is “understandable and teachable.”

That brings us back to the problems mentioned in the previous slide.

Quantitative literacy must be “understandable and teachable.”

Whether QL is understandable and teachable depends critically on the content of QL.

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QL
Numbers in Context

“The essence of QL is to use mathematical and logical thinking in context.” Lynn Steen 2004

QL must have defining core concepts that are

- based on the role of context in arguments
- mathematically sound
- understandable by students and faculty
- useful to students in their everyday lives
- teachable by non-math faculty.

IF QL is to be interdisciplinary, it may vary by discipline (as does writing).
But there must be some things that are **COMMON** to all disciplines (as does writing).

Carleton College recently received a grant to develop Quantitative Literacy across the curriculum. According to Neil Lutsky, the faculty at Carleton discussed QL across the curriculum for 2 years. They concluded that a focus on argumentation was essential of QL was to be interdisciplinary. I had invited Neil to say a few words on this matter, but he is reviewing student portfolios tonight. He and his colleagues are trying to analyze the level of quantitative reasoning used in these portfolios. I see the work at Carleton College as being on the cutting edge of quantitative literacy.

So what COMMON things must QL address to be accepted by faculty across the curriculum?

1. QL must deal with arguments.
Arguments are common – but the arguments must be inductive.
They cannot be limited to just the kind of math: deductive arguments, right-wrong with answers in the back of the book.
And the statistics involved aren’t just typical word problems.
The numbers are typically used as evidence for non-statistical conclusions.

2. QL must focus on arguments encountered in our everyday lives.
We cannot focus on things that are seldom if ever encountered.
Health issues, financial issues and social issues: everyday arguments.
These issues are central to our life as citizens in a democratic republic.

3. The core ideas of QL must be teachable by non-math faculty.

4. 4. The core ideas of QL must involve principles that are mathematically sound.

> Consider four concepts that I believe meet these four requirements >

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**QL:
Four Core Concepts**

Whether QL is a separate course or is infused in other courses, it must have core concepts.

Here are some good candidates:
Four key math tools that control for context:

1. Arithmetic comparisons (% more than)
2. Ratios (percentages, rates, probability)
3. Comparisons of ratios (likely, prevalent)
4. **Standardizing (compare apples w. apples)**

Here are my candidates for four core concepts in quantitative literacy. They are involved in all kinds of arguments on a daily basis. They are simple enough to be teachable by all faculty.

#1 click: Arithmetic comparisons. Six is three times two. That sounds easy

#2 click: Ratios such as percentages and rates.

Ann Landers asked parents if they had it to do over, would they have kids.

70% said No. Ratios can be simple but their interpretation may not be obvious.

#3 click: Comparisons of ratios.

Men are three times as likely to die accidentally as are women.

#4 click: In making comparisons between groups, the groups should be similar. More on this one later.

Now you have an overview of the four topics.

The rest of my talk is to present details on these four concepts.

> Let's start with #1: Arithmetic comparisons >

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**#1: Numeric Comparisons
Control For Context**

Qualitative vs. quantitative

- Napoleon was shorter than many French soldiers
- Napoleon 4" shorter than average French soldier

- Women live longer than men
- Women can expect to live 7 years longer than men

If interest rates increase from 1% to 2%.

- Double (two times as much as)
- 100% increase (100% more; 1 times more than)
- 1 percentage point increase **Not a 1% increase!**

Suppose someone says, “Napoleon was short.” This doesn’t give much information.

A qualitative comparison is better.

But a quantitative comparison is still better.

Quantitative comparisons measure relative to some standard or basis.

But comparison can be tricky.

When interest rates increase from 1% to 2%, do they increase by 1%?

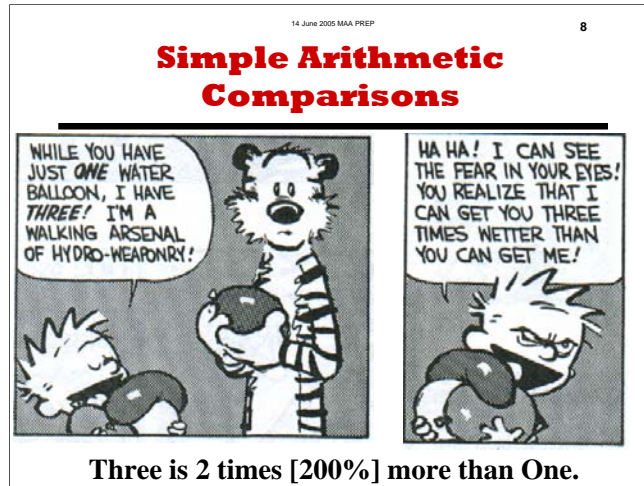
Yes, they double.

Yes, they increase by 100%.

#1 click: No, they do not increase by 1%.

They increase by 1 percentage point.

> Consider another mistaken arithmetic comparison in the comics >



Three water balloons for Calvin.

One water balloon for Hobbes.

“Three times as many” means “three times as wet.”

But not “three times wetter.”

#1 click: Three is 2 times more than 1

> Now let’s turn to the second core concept: Ratios. >

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#2:
Ratios Control For Context

Part-whole ratios are conditional probabilities.

- $P(B|A)$

Algebra is clean and unambiguous.
Ordinary English is messy and ambiguous
But students speak English – not Algebra

Q. Can these both be true for the same group?

1. Unemployment is up Number is up
2. Unemployment is down Rate is down

Ratios are ordered relationships just like subtraction and division.

Kids find addition and multiplication easier than subtraction and division.

Addition and multiplication are the same regardless of order.

Subtraction, division, comparisons and probabilities are order dependent.

To be interdisciplinary, this subject should be taught using ordinary English.

How can unemployment be both up and down?

#1 click: As a number, unemployment is up (as population increases).

#2 click: As a rate, unemployment is down.

> Consider descriptions of percentages and rates using ordinary English >

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#2

Ratios Control For Context

Q1. Are these percentages the same?

1. The percentage of men **WHO ARE** runners
2. The percentage of men **AMONG** runners

Q2. Are these rates the same?

3. The women's death rate
4. The death rate of women
5. The rate of death among women
6. The women's rate of death

Question: Are these the percentage statements the same?

Answer: Absolutely not.

Do you know the rules for identifying part and whole in ordinary English?

If not, how can you teach students to decode such statements.

Q2. Are these rate statements the same?

Answer: Yes.


This is not rocket science, but it takes practice to identify part and whole.

> Consider a percentage encountered in every drug store in the US. >

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Q/L: Interpreting Medical Tests
99.9% accurate!

- Greater Than 99.9% Accurate
Reliable as Tests Used by Doctors and Hospitals
- Confidential and Anonymous
- Results 24 Hours a Day
- One Spot™ Technology



HIV - 1 TEST SYSTEM
For the Detection of Antibodies to HIV-1

Self-test medical kits are available in drug stores.

Anyone can buy a kit for self-testing on HIV for under \$50 in any drug store.

You take a blood sample and mail it in to a lab.

Later, from a pay phone at midnight, you call an 800 number.

You give them the code number on the side of the box.

You hear one of two words: “positive” or “negative.”

Suppose the recording says, “POSITIVE”.

Do you have HIV?

What’s the chance that you don’t.

You look at the 99.9% accuracy on the side of the box.

> How do you interpret this number?>

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**“99.9% Accurate”
Statistical Prevarication:**

Q. Is this accuracy in prediction?

- **99.9% of those testing positive have HIV?**
NO!

“99.9%” involves confirmation, not prediction

Confirmation:

- **Of those with HIV, 99.9% test positive**

Prediction is typically a different number:
Suppose that 0.1% of a population have HIV.
50% of those testing positive, will have HIV

Prevaricating is telling a half truth. It is not lying.

The 99.9% involves a statistical prevarication.

See my paper, Statistical Prevarication: Telling Half Truths w/o Lying

99.9% accurate. Does this mean 99.9% of those testing positive have HIV?

#1 Click: Most users think so. But that conclusion is false.

#2 Click: Accuracy in this context means accuracy in confirmation.

#3 click: 99.9% accuracy means 99.9% of those with HIV test positive.

This test is very accurate in confirming what is already known.

#4 click: Prediction is typically a very different number.

If one person in a thousand has HIV,

then this test predicts that 50% of those testing positive will have HIV.

To be quantitatively literate, adults must know this difference.

This finishes our presentation of the second core concept: ratios.

> Now let's turn to the third core concept: comparisons of ratios >

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#3: Comparisons of Ratios
Control For Context Two Ways

Is marijuana a gateway drug to heroin?

1. 90% of heroin addicts first used marijuana
2. 99% of heroin addicts first used milk

Are men psychologically stronger than women?

3. Widows are more likely **AMONG** suicides than widowers [are].
4. Widows are *less* likely **TO** commit suicide than widowers [are].

90% of heroin users first used marijuana.

Is this strong evidence for criminalizing marijuana as a gateway drug?

A high percentage may be strong evidence for causation.

But a comparison of ratios can take into account the influence of context.

#1 click: 99% of heroin addicts first “used” milk.

If we criminalize marijuana, are we going to criminalize milk?

This 99% statistic is weak evidence for causation.

#2 click:

Consider this thesis: Men are psychologically stronger than women.

The people involved are survivors from the death of their spouse.

Widows are women. Widowers are men.

The statistic: Women are more likely among suicides than widowers.

Doesn't this support the claim that men are psychologically stronger?

No it doesn't.

There are more widows than widowers.

#4 click: This is the comparison we need.

Widows are less likely to commit suicide than are widowers.

Small changes in syntax can generate big changes in semantics.

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**Augsburg StatLit Project:
Web-based Tools**

Simple Surveys: www.StatLit.org/Survey

Grammar Checker: www.StatLit.org/GC
User Goal: To read a table of rates and percentages, to decode the meaning, and to write a single sentence in ordinary English that describes a single ratio or compares two ratios.
Program Goal: To decode a user's sentence, identify errors and give helpful error messages.

Have your students try the simple survey in reading simple tables of percentages.

In my last class, only 3 students out of 20 got all the questions correct.

If you are very interested in helping students use ordinary English to describe and compare ratios,

check out the online grammar checker.

This program is absolutely amazing.

It is not a pattern recognition program. It diagrams sentences.

It uses ordinary English syntax to decode the underlying semantics.

Half of students find this program “very helpful”. Here are some student comments:

I need practice and this program helped me practice.

Very helpful in learning material.

The program has, for me, been the key to really understanding the material.

It's a unique way of doing homework to study.

Practice! Practice! Practice! It was great to be able to practice and get feedback.

Helpful when professor not available.

It gave me more of an idea on structure.

Helps me learn the grammar more directly

> Let's take a peak at this brand new program >

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**#4: Standardizing Ratios
Controls For Context**

Once you have ratios (percentages, rates or averages) or comparisons of ratios, many students mistakenly think no more can be done.

Standardizing takes into account the influence of confounders on ratios.

Standardizing links mathematics, confounding and context in ways that everyone should know.

Standardizing involves multivariate thinking.

Most students think that if you compare rates, you are done. Yet the hospital with the highest death rate is often the best. They have no idea that death rates can be – and should be – standardized.

Q. So what is “Standardizing”?

This is a term taken from demographics.

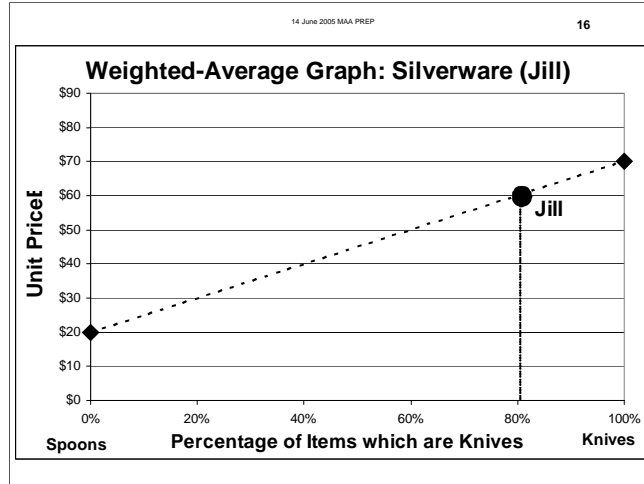
As the US population ages, the prevalence of age-related diseases and conditions increases.

To take into account age, the population distribution by age is “standardized.”

Standardizing is not a simple division like a fraction or a ratio comparison. Standardizing is not a simple arithmetic operation like a percentage change.

Every student in GST 200 Quantitative Literacy at Augsburg College studies this topic.

> Consider a very simple example involving shopping for silverware >



Jill is shopping for silverware at an OUTLET STORE
She is buying knives and spoons.

At this outlet store, she can buy knives at \$70 each.
At this outlet store, she can buy spoons at \$20 each.

Study this graph carefully.

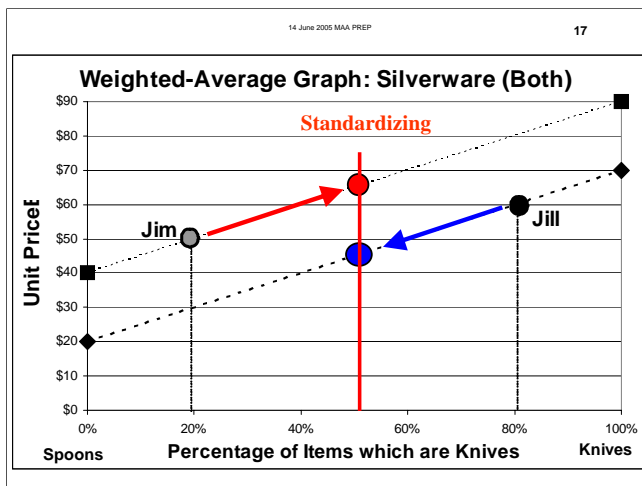
This graph is essential in understanding the effect of context on numbers.

On the horizontal axis, there are two natural groups: knives and spoons.
There are no 50% knives or 80% spoons.

Her average cost is a weighted average of these two prices.
If she buys half of each, her average cost is \$45.

Suppose she buys 8 knives and 2 spoons (80% knives).
Her average cost is \$60 at this outlet store.

>Her friend Jim says she is wasting time driving to an outlet store >



Jim buys his silverware at a local mall.
 Jim paid \$50 on average for his silverware at the local mall.
 Jim's average cost (\$50) is less than Jill's average cost (\$60).
 But these are weighted averages.
 Look at the difference. Jim bought fewer knives and more spoons.

#1 Click: Standardizing involves using the same mix for both cases.
 What would each person have paid on average if they had the same mix?

- #2 click Suppose we standardize on 50%.
- #3 click: Jim's average price would have increased.
- #4 click: Jill's average price would have decreased.

Jim's standardized price is bigger than Jill's.
 Students find this explanation intuitive and satisfying.
 This reversal is called Simpson's Paradox.
 It is quite common. See our paper on Simpson's Paradox in NAEP Data.

Standardizing controls for the influence of a confounder on averages.
 Standardization involves a simple case of multivariate regression.
 > Now let's try a real problem involving a controversial social issue >

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**#4: Numbers in Context:
Multivariate Thinking**

Let's try an example in Public Affairs:

Average family income:

- \$41,000 for US white families
- \$25,000 for US black families
- \$16,000 is the black-white income gap

Is this evidence of structural racism in America?

Quantitative Literacy does not attempt to prove that A causes B.

Quantitative Literacy focuses on evaluating the quality of the number used in that argument.

Is the number spurious?

Consider the \$16,000 black-white income gap in the US.

It could be causal – structural racism.

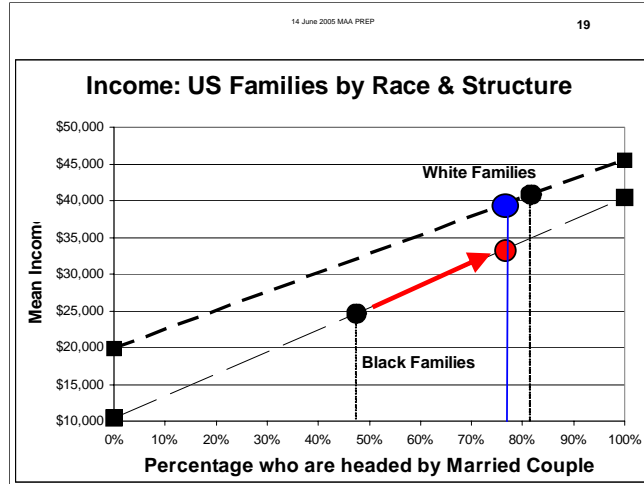
But it could be tangled up with related factors.

Q. What related factor could influence family income?

Let's consider family structure.

Married couples can have more wage earners than single-parents.

> Consider standardizing these incomes on family structure >



Consider the underlying data for families.

Married couples are on the right (they are the knives).

Single-parents are on the left (they are the spoons).

For whites (top line), married couples average \$45K; single-parents average \$20K
82% of white families are headed by a married couple.

For blacks (bottom line), married couples average \$40K; single-parents \$10K
48% of black families are headed by a married couple.

Average family income is \$41K for whites and \$25K for blacks.

\$16K is the black-white income gap that might appear in the news.

Now let's standardize these family incomes.

#1 click: 78% of all US families are headed by a married couple.

#2 click: White family income would decrease to \$39K

#3 click: Black family income would increase.

#4 click: Black family income would increase to \$33K.

The standardized difference is 6,000: \$39,000 – 33,000.

Of the original difference (\$16,000), \$10,000 is explained by family status.

63% of the black-white income gap is explained by differences in family status.

Now the evidence is weaker for racism: \$6K instead of \$16K

To understand numbers in context, students must understand standardization.

> Now let's look at some examples of un-standardized comparisons in the news >

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**#4: Numbers in Context:
Seeing Confounding**

Mexico has better medical care than the US.

- Death rate in Mexico: 5 per 1,000 population
- Death rate in US: 8.7 per 1,000 population

Utah schools (227) better than Oklahoma (225)
 NAEP score: 4th grade Math in 2000n.
 OK higher than UT for low-income kids & for high-income kids. OK had more low-income kids

Typically, readers of quantitative data do not have all the data needed to standardize.
 But they must think creatively about what related factor can affect an association.

- Q. Does Mexico have better health care than the US.
 Mexico's death rate (5 per 1,000) is less than the US death rate (8.7 per 1,000).
 Q. So do you think Mexico has better health care than the US?
 If not, what else could explain the difference in death rates?

Here is where readers must think creatively.
 The answer is not typically stated in the argument.

One possibility is AGE.
 Older people are much more likely to die than are younger people.
 The US has a higher percentage of Seniors than Mexico.
 Mexico has a much higher birth rate than the US.
 Mexico has a much younger population than the US.

If Mexico had the same age distribution as the US,
 then the standardized death rate in Mexico might be higher than in the US.
 If so, we would have yet another instance of Simpson's Paradox in our data.

=====

Does Utah have better schools than Oklahoma?
 Utah students scored higher on the 4th grade NAEP test than did Oklahoma students.
 But what else might be confounded – tangled up – with school performance?
 Perhaps family income.

Kids from wealthier families may have more access to educational materials & programs.
 If Utah has a higher percentage of kids from high-income families, that might explain the association.
 And in this case it does.
 Oklahoma kids scored higher than Utah kids in both subgroups.
 Once again, we see Simpson's Paradox in our data.

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Quantitative Literacy
Gina Kolata, NY Times

Beyond arithmetic and geometry,
quantitative literacy also requires
logic, data analysis and probability...

It enables individuals to analyze evidence,
to read graphs, to understand logical arguments,
to detect logical fallacies,
to understand evidence and to evaluate risks.

Quantitative literacy means **how to reason**
and **how to think**.

Gina Kolata is a well-respected science reporter for the New York Times. I have presented four concepts that I see as core to Quantitative Literacy. But there is much more that can be said about numbers. We have talked about one alternate explanation for a number. We have talked about just about confounding.

> Let's look at numbers in context from a broader perspective.

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Numbers in Context
Take “CARE”

Associations have many explanations:
Causation [of outcome by predictor]

- **C**onfounding (may be a common cause)
- **A**ssembly (people choose the stats presented)
- **R**andomness (more likely in small samples)
- **E**rror/Bias (sampling bias, subject bias, etc.)

To support causation, one must be able to eliminate alternate explanations (CARE).

Associations have multiple explanations. Consider this association:

Women who drink green tea weigh less than women who don't.

The association may involve causation.

Maybe Green tea has some chemical property that suppresses weight gain.

But as mathematicians and statisticians, our best advice must be

TAKE CARE. There are many alternate explanations for a number.

Real numbers exist in a context. They are not Platonic forms.

*C. The association may involve confounding.

Maybe most of the women drinking green tea are short Orientals.

The shorter height explains why they weigh less – not the Green tea.

*A. The association may involve Assembly.

How did they define “women who drink Green Tea”?

Drink 10 cups a day? Drink Green tea for 10 years?

Bottom Line: Definitions can be assembled in many ways.

The association may be due to the choice of definitions – to assembly.

*R: The association may be due to randomness.

The smaller the sample, the easier it is for randomness to explain the result.

*E: The association may be due to error or bias.

Maybe the subjects self-reported their weight. Did they fudge?

To understand numbers in context, to understand the influence of context on numbers,

People must understand all four kinds of influence on numbers.

To read more on this I strongly recommend Joel Best's books,

“Damned Lies and Statistics” and “More Damned Lies and Statistics”

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QL Has a Bright Future

If QL can agree on some core QL ideas that

- are common across the curriculum,
- focus on arguments in everyday life,
- relate to context, and
- enhance students' critical thinking

then Quantitative Literacy will be

valued, respected and accepted in academia.

Working together, we can make it happen!

I strongly believe that Quantitative Literacy has a bright future.
 We have a big job ahead of us. We are trying to change higher education.
 We are trying to heard cats – to shape fog.

But we have the tools.

If we can agree on some core QL ideas and concepts,

Ideas and concepts that

- Are common across the curriculum,
- focus on arguments in everyday life,
- relate to context,
- enhance students' critical thinking,

THEN Quantitative Literacy will be

#1 click: VALUED, RESPECTED and ACCEPTED in academia.

#2 click: Working together we can make it happen.

Thank you.

References atwww.Augsburg.edu/StatLit

Schild, Milo (2005a). *Statistical Prevarication: Telling Half Truths without Lying*. IASE conference Sydney

Schild, Milo (2005b). *Statistical Literacy: An Evangelical Calling for Statistical Educators*. ISI Au.

Schild, Milo (2005c). *Information Literacy, Statistical Literacy and Data Literacy*. IASSIST IQ Quarterly.

Schild, Milo (2004). *Statistical Literacy and Liberal Education at Augsburg College*. AAC&U Peer Review

Terwilliger, Jim and Milo Schild (2004). *Frequency of Simpson's Paradox in NAEP Data*. AERA