Statistical Literacy and Chance

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Key Topics Involving Chance

1. From prevalence to probability
2. Random sampling vs. random assignment
3. Law of Very Large Numbers for Rare Events
4. **Law of Large Numbers for averages**
5. **Confidence Level and Conf. Interval**
6. **Hypothesis Tests and Statistical Significance**
7. Confounder influence on statistical significance

#1: From Prevalence to Probability

Suppose: 2% of adults developed AIDS last year.

- The chance that a randomly-selected adult **developed** AIDS **last year** is 2%.
- The chance that a randomly-selected adult **develops** AIDS is 2% **per year**.
- The chance that a randomly-selected adult **will develop** AIDS **next year** is 2%
- The chance that **you** (a random adult) will develop AIDS **next year** is 2%.

#2: Random Selection vs. Random Assignment

Random Selection and Random Assignment are:
- totally haphazard or highly controlled?

Random assignment means 'no control' by
- subject, researcher or both?

Random assignment statistically controls for:
- all confounders or just existing confounders?

Random selection statistically controls for:
- all confounders or no confounders?

#3A: Law of Extremely Large Numbers

The unlikely is almost certain with enough tries.

What is chance of 8 heads in a row for fair coin? Question is ambiguous.

If prospective (looking at next 8), very unlikely:
- 1 chance in 2^8: (256)

If retrospective (cherry-picking), very likely:
- 63% chance in 256 samples, 90% in 589.

#3B: Trojan Horse of Very Large Numbers

Trojan Horse: A case where looks deceive.

Q. Does a very large sample size (>10,000) all but guarantee a result is statistically significant?

Yes if statistic is a quantity shared by all.
E.g, average height or average income.

No if statistic is a outcome shared by just a few.
E.g., a rare event or low rate.

#4: Law of Large Numbers

As size of a random sample increases, the sample average is less likely to deviate (likely to deviate less) from population mean.

If N is population size and n is sample size, then the Margin of Error is proportional
- to the SQRT{[1+(n/N)]/n} for N ~ n.
- to the SQRT(1/n) for N >> n.

Margin of error is independent of population size if population is much larger than sample.

#5: Confidence Intervals and Confidence Level

95% of all 95% Confidence Intervals
contain the population parameter.

We have a 95% chance of randomly-selecting a 95% Conf. Interval that contains the parameter.

We are 95% confident that a randomly-selected (RS) 95% Conf. Interval contains the pop. parameter.

We are 95% confident (in betting) that this RS 95% Conf. Int. contains the pop. parameter.
See Schield (1997). *Interpreting Statistical Confidence*

#6a: Hypothesis Testing & Conditional Reasoning

Outcome is unlikely (statistically significant):
- IF due to chance. [Frequentist]
- TO BE due to chance. [Bayesian]
- due to chance. [Statistical prevarication]

Q. What is a hypothesis test is:
- a test of the null hypothesis?
- a test of the alternate hypothesis?
- a test of the sample statistic?

#6b: Hypothesis Testing & Bayesian Reasoning

If truth of alternate hypothesis is
- > 50%, then P(Null is true) < P-value.
- = 50%, then P(Null is true) = P-value.
- < 50%, then P(Null is true) > P-value.

See Schield (1996) Bayesian Inference in Hypothesis Testing

Historically, science dealt with alternates having strong evidence so a cutoff of 5% was OK.

Today, science deals with alternates having weak evidence so cutoff must be less than 5%.

#7: Confounding and Statistical Significance

In an observational study, a confounder can
- make a stat. sig. relation stat. insignificant.
- make a stat. insignificant relation stat. sig.

Showing the influence of context (confounding) on statistics requires multivariate thinking.

Failing to teach this to students dealing with observational data is professional negligence.
See Schield (2004) *Three Graphs to Promote Statistical Literacy*

#7: Confounder Influence on Statistical Significance