Important Mathematical Concepts for Numeracy

This paper is based on an informal talk given by Bernie Madison at MAA 2005

First I would like to say something about the traditional college algebra course and what I think it is as far as developing any kind of foundation for QL.

The concepts of the college algebra course are very, very difficult to ferret out, primarily because they are couched in abstract algebra terms. So the students don't have a conceptual framework on which to hang all of this stuff. And there are few connections at all to the real world and these are artificial. Consequently the course gets a lot of bad press. I don't know if you saw the article in the New York Times a few weeks ago (December 12, 2004) entitled *The Last Time You Used Algebra Was…* by Donald G. McNeil, Jr. You should read it because it's damning! The gist of the article is why bother, why punish people on 'the cross of X and Y'? The course is a very difficult teaching assignment partly because it is so unpopular. It's got some fairly complex methods and algorithms. And it's often taught – at least at my place – by graduate students and by adjuncts, people who are perfectly willing – and often only able -- to deal with the course in that kind of mechanical way. Consequently the course has degenerated into mechanics. So, it's very difficult to see how it connects to the real world.

Over the last several years, I've been looking at quantitative literacy from several different points of view. And finally, last year, I decided that I needed to take what I had learned to the classroom. I didn't teach a college algebra course; I'm actually teaching a course that has college algebra as a prerequisite. I checked on how much algebra the students knew and they knew very little – practically nothing. They couldn't write the equation for a line, they couldn't graph a straight line – at least as far as I could tell.

But here is what I did in my class. I didn't use any textbook. I used only newspaper and magazine articles – mostly ones that I have collected over the past four years. Half of the 26 students were journalism majors; the others were majors in the arts and humanities. Next semester I'm going to do this again with all journalism majors. The students were asked to look at the newspaper and bring in articles. They had to bring in something and explain the mathematics and statistics of it. We did that the first part of each class. By using news articles the connections to the real world were automatic. This – the news articles – is the real world, so I never had that question. The students never asked me, "Why do we have to do this?" The articles were in the morning newspaper. So, consequently I had that the problem of connecting to the real world satisfied. All I had to do was figure out how to teach the students – rather convince them to learn – to analyze those articles, take out the mathematics and statistics in those articles, use the mathematics and statistics, check it, do further analysis and then reflect back into the articles.

This combination of word and number is deadly difficult, as even the traditional college algebra sections on word problems have told us. Very few students can do what I was trying to get the students to do. Check your math majors at the ends of their programs, check your graduate students, check your colleagues and see which of them can do this. It is very difficult; very few even try and do it.

So here are some glimpses at what I did, and I've done quite a bit of work with this. I want to share with you in about 5 minutes some of the lessons I've ferreted out of this.

Here are what I think are important mathematical concepts for numeracy. Now I'm using the international word for quantitative literacy because I used these slides in Canada last month at Banff at the PIMS[1] conference on "Numeracy and Beyond." So just read Quantitative Literacy where you see Numeracy.

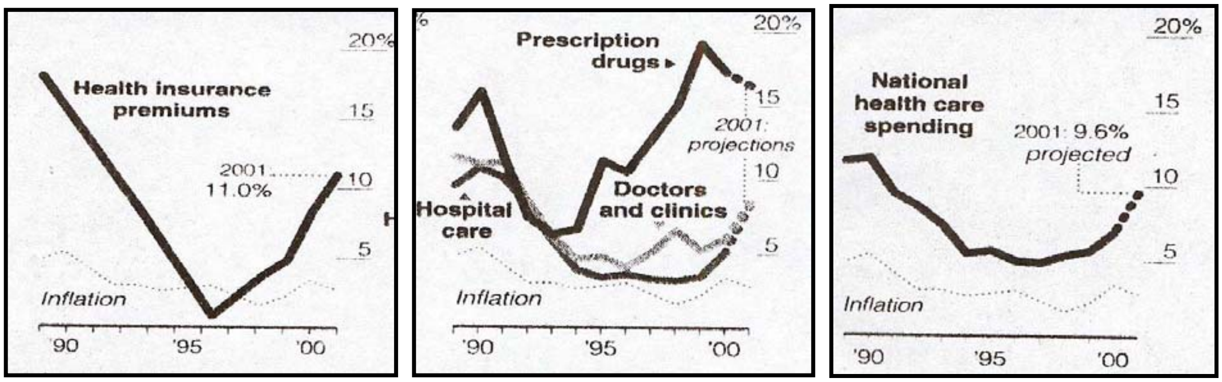
[1] www.pims.math.ca/birs/workshops/2004/04w5044/

IMPORTANT MATHEMATICAL CONCEPTS:

- Rates and rates of change: Absolutely essential; they occur in many newspaper articles every day.
- A 'percent' – what is it? And percent change. You can actually make the meaning of percent rigorous. Last week, I listened to a distinguished mathematician (Hung-His Wu from UC Berkeley) tell me exactly what a percent was and put it on a rigorous complex fraction basis. I thought it was wonderful but I would never dare share that with my students. But they have real trouble with percent – particularly percent change. And they have particular trouble with small numbers and large numbers. Small fractions like 0.05% percent are really tough and large fractions like 340% are also.
- [Times less]. One of our local supermarkets was advertising its' meat as having "five times less sodium than Wal-Mart." Now I keep asking my students, "What does 'five time less' mean." I've asked my wife that, I've asked my daughter that – and nobody has really told me what that really means. I'm going to the supermarket next week and ask them.
- Graphs of the first and second derivative. They do come up in the newspaper and I'll show you a couple. People don't recognize what they are.
- Linear and exponential rates of growth: Absolutely essential.
- Accumulation – believe it or not. Notice that these – rates of change and accumulation – are the two main ideas in calculus.
- Installment loans, savings and weighted averages: they keep coming up. Students need to learn to handle them.
- Indexes and Condensed Measures: this is something we don't teach at all. I wager that they are very few people in this room who can give me a rigorous definition of an index. I've thought about it and tried to explain it to my class. 'Condensed measures' is the term that I've heard – that's used in the literature – and they are just limited measures of some kind of variation, like 'poverty line.' But there are certain things that are called indexes but are not according to the definition I know – like the Dow Jones Industrial Average (DJIA) and the Body Mass Index.
- Estimation: In many cases, estimation is the most important lesson of the day. Estimation has become incredibly important, more so because of computers, but very sophisticated.
- Plane geometry. My students didn't know any plane geometry by the way. They didn't know the simplest volume and radius formulas. They didn't remember them, because they hadn't used them in real life.
- Graphical production and representation: I'll show you some graphs in a minute.
- Probability: single and conditional, the idea of risk and odds. Students didn't know what odds meant. They didn't know how they could combine odds; they didn't know what risk was. They had never even thought that they could understand it.

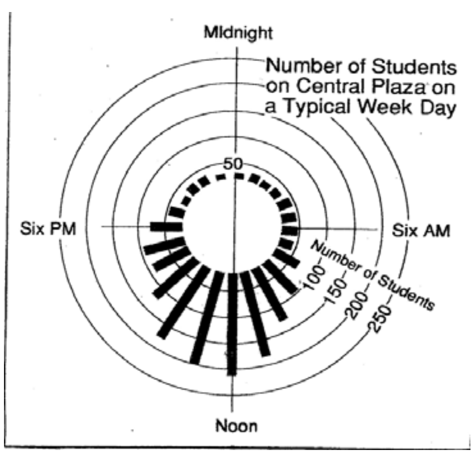
Graphs: One of the things that we spent a lot of time on was looking at graphs out of the newspaper and trying to figure out if they were the same graph that we saw in an algebra book. They didn't look at all like the graphs in an algebra class. In fact to even call them the same name seemed kind of silly. There is so much information in some of these graphs and the presentations are so unorthodox that students see these newspaper graphs as something different from what we deal with in geometry classes or algebra class. We definitely need to make those connections.

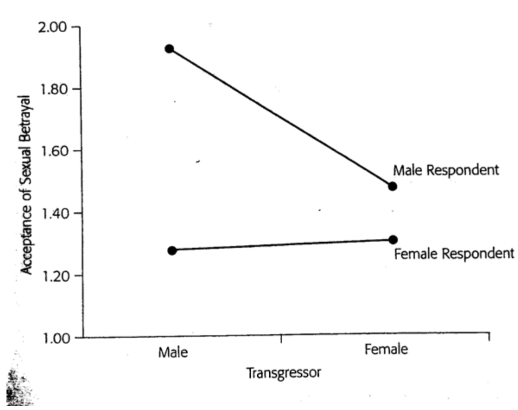
- Second Derivative Graph: Do you want to see a graph of the second derivative? It's called "The Rise in Spending." It's in the *New York Times* so it must be true.



In fact, this is a graph of the second derivative. This is spending in health insurance, prescription drugs, and national heath care spending. It is called rise in spending and yet those graphs don't rise. And students don't really understand this. What they've got to do is somehow understand that this is the second derivative of costs, but not in those sophisticated terms. What I had my students do was to assume that insurance premiums cost $200,000,000 in 1990, use this graph of the annual percentage increases and produce a graph of that cost over this 10 year period. You've got to take the rates of change and "fold" them back into the behavior of the actual costs. The students didn't have a clue of how to do this. And now they understand it – actually, I believe they do. At least, they understood it last week. Unless they practice it they will forget it, just like factoring polynomials.

- Circle Clock Graph: Now this is one I had never seen before. I understand from my colleagues in statistics that it's not all that unusual. One of the students brought this in. It's a graph based on a plot – a bar graph – with the length of the bars placed over the hours of the day. It's a good idea, I thought. It's a very good idea. It's very visual. It's very informative if you look at it.





- There were some others that students brought in that I thought were a little less informative than this clock-face-based graph. Here's one that is very interesting because it came out of one of the student's textbook in sociology. They didn't understand why it was a graph. Why is this a graph? It has to do with the acceptance of sexual betrayal. And it shows that males are very accepting of this sexual betrayal and females are not. There are four data points there but there are two line segments. And the students wanted to know, "Where did those lines come from?" I thought it was a wonderful question because when we draw lines in a college algebra class, they mean something. In this case they the lines are just for some visualization -- probably.

So my point about this is that our graphs in algebra don't look like those in the news. Now that doesn't make our graphs wrong; it doesn't make graphs in the news wrong either. Sometimes they are wrong; sometimes they are inconsistent. We need to build connections between what we do and what our students meet out there in the real world. I've been trying to do that.

I would love for you to read the student comments about this course. These are students who, as a group, disliked mathematics. This was their last math course; they will never see math again, not in a college math classroom. And several students wrote comments much the same as one student who wrote, "This is the first time I've tried to learn mathematics; it's the first math course that didn't bore me to death." But it's unfair to take their comments like that at full face value because I was catering to them. I fully admit that I was catering to the students.

The mathematics out of these articles was not the primary difficulty for them. They were much better at doing the mathematics than they were at deciding what mathematics was present and then reflecting back into the articles after doing some math or statistics. That makes it an elementary math course I guess. But let me tell you something. It is not an elementary course. Now whether this is mathematics or not, I won't argue with you. I think it is worthwhile. I think it uses mathematics. I think it can be very valuable to mathematics because it makes mathematics popular and we may get students to keep looking at mathematics out there in the real world. They've got to develop the habit of looking at mathematics and confronting it in every day life. They've got to practice this. Habits are developed out of practice. Unless we give them the opportunity – the means – to develop these habits, they will never do it. "The Last Time You Used Algebra Was… " – read that article. These students hadn't practiced algebra since they left the algebra course in high school or the college algebra course a few semesters earlier. So consequently they weren't able to recall it and use it without some reason to do so. So we've got to give them a context in which they can practice.

And as was said before, mathematical educators can't do this alone. We've got to ask our colleagues over in sociology and other places to do it. But we're doing that at our place too.

OK. Now we've got time for questions.