THE CASE FOR QUANTITATIVE LITERACY

The world of the twenty-first century is a world awash in numbers. Headlines use quantitative measures to report increases in gasoline prices, changes in SAT scores, risks of dying from colon cancer, and numbers of refugees from the latest ethnic war. Advertisements use numbers to compete over costs of cell phone contracts and low-interest car loans. Sports reporting abounds in team statistics and odds on forthcoming competitions.

More important for many people are the rapidly increasing uses of quantitative thinking in the workplace, in education, and in nearly every other field of human endeavor. Farmers use computers to find markets, analyze soil, and deliver controlled amounts of seeds and nutrients; nurses use unit conversions to verify accuracy of drug dosages; sociologists draw inferences from data to understand human behavior; biologists develop computer algorithms to map the human genome; factory supervisors use "six-sigma" strategies to ensure quality control; entrepreneurs project markets and costs using computer spreadsheets; lawyers use statistical evidence and arguments involving probabilities to convince jurors. The roles played by numbers and data in contemporary society are virtually endless.

Unfortunately, despite years of study and life experience in an environment immersed in data, many educated adults remain functionally innumerate. Most U.S. students leave high school with quantitative skills far below what they need to live well in today's society; businesses lament the lack of technical and quantitative skills of prospective employees; and virtually every college finds that many students need remedial mathematics. Data from the National Assessment of Educational Progress (NAEP) show that the average mathematics performance of seventeen-year-old students has risen just one percent in 25 years and remains, at 307, in the lower half of the "basic" range (286–336) and well below the "proficient" range (336–367). Moreover, despite slight growth in recent years, average scores of Hispanic students (292) and black students (286) are near the bottom of the "basic" range (NCES, 1997).

Common responses to this well-known problem are either to demand more years of high school mathematics or more rigorous standards for graduation. Yet even individuals who have studied trigonometry and calculus often remain largely ignorant of common abuses of data and all too often find themselves unable to comprehend (much less to articulate) the nuances of quantitative inferences. As it turns out, it is not calculus but numeracy that is the key to understanding our data-drenched society.

Quantitatively literate citizens need to know more than formulas and equations. They need a predisposition to look at the world through mathematical eyes, to see the benefits (and risks) of thinking quantitatively about commonplace issues, and to approach complex problems with confidence in the value of careful reasoning. Quantitative literacy empowers people by giving them tools to think for themselves, to ask intelligent questions of experts, and to confront authority confidently. These are skills required to thrive in the modern world.

A Brief History of Quantitative Literacy

Although the discipline of mathematics has a very ancient history—both as a logical system of axioms, hypotheses, and deductions and as a tool for empirical analysis of the natural world—the expectation that ordinary citizens be quantitatively literate is primarily a phenomenon of the late twentieth century. In ancient times, numbers, especially large numbers, served more as metaphors than as measurements. The importance of quantitative methods in the lives of ordinary people emerged very slowly in the middle ages as artists and merchants learned the value of imposing standardized measures of length, time, and money on their arts and crafts—for example, in polyphonic music, perspective drawing, and double-entry bookkeeping (Crosby, 1997).

In colonial America, leaders such as Franklin and Jefferson promoted numeracy to support the new experiment in popular democracy, even as skeptics questioned the legitimacy of policy arguments based on empirical rather than religious grounds (Cohen, 1982). Only in the latter part of the twentieth century did quantitative methods achieve their current status as the dominant form of acceptable evidence in most areas of public life (Bernstein, 1996; Porter, 1995; Wise, 1995). Despite their origins in astrology, numerology, and eschatology, numbers have become the chief instruments through which we attempt to exercise control over nature, over risk, and over life itself.

As the gap has widened between the quantitative needs of citizens and the quantitative capacity of individuals, publications about "math anxiety" and "math panic" have raised public awareness of the consequences of innumeracy (Buxton, 1991; Paulos, 1988, 1996; Tobias, 1978, 1993). At the same time, publications such as Edward Tufte's extraordinary volumes on the visual display of quantitative information reveal the unprecedented power of quantitative information to communicate and persuade (Tufte, 1983, 1990, 1997). We see the results every day, both good and bad, in the widespread practice in newspapers of using charts and graphs as the preferred means of presenting quantitative information.

In 1989 the National Council of Teachers of Mathematics (NCTM) responded to the changing mathematical needs of society by publishing standards for school mathematics that called for all students to learn rich and challenging mathematics. Subsequently, other standards documented the role of quantitative methods in education (e.g., science, history, geography, social studies) and careers (e.g., bioscience, electronics, health care, photonics). In April 2000 NCTM released a much-anticipated update of its standards for school mathematics (NCTM, 2000). These standards and their interpretations in state frameworks, textbooks, curricula, and assessments have engendered considerable public debate about the goals of education and about the relation of mathematics to these goals.

In recognition of the increasing importance of quantitative literacy in the lives of nations, government agencies that monitor literacy divided what had been a single concept into three components: prose, document, and quantitative literacy (Kirsch and Jungeblut, 1986; NCES, 1993; OECD, 1995, 1998). Similar awareness led many liberal arts colleges to infuse quantitative methods into courses in the arts and humanities (White, 1981). At the same time, economists expanded the traditional "3 R's" requirement for employment (reading, 'riting, 'rithmetic) to encompass five additional competencies: resources, interpersonal, information, systems, and technology (SCANS, 1991). More recent publications have examined the role of quantitative literacy in relation to the changing economy (Murnane and Levy, 1996), the expectations of college graduates (Sons, 1996), the perspectives of professionals in a variety of fields (Steen, 1997), and the demands of the high-performance workplace (Forman and Steen, 1999).

The footprints of quantitative literacy can be found throughout these publications, but not clarity about its meaning. These sources reveal more confusion than consensus about the nature of quantitative literacy, especially about its relation to mathematics. They echo the historical dichotomy of mathematics as academic and numeracy as commercial, and pay at most lip service to the role numeracy plays in informing citizens and supporting democratic government. What we learn is that although almost everyone believes quantitative literacy to be important, there is little agreement on just what it is.

Mathematics, Statistics, and Quantitative Literacy

In the beginning, grammar schools taught arithmetic and colleges, mathematics. As secondary schools became the transition from grammar school to college, courses in algebra, geometry, trigonometry, analytic geometry, and even calculus created a highway that led increasing numbers of students directly from arithmetic to higher mathematics. At the same time, mathematics itself expanded into a collection of mathematical sciences that now includes, in addition to traditional pure and applied mathematics, subjects such as statistics, financial mathematics, theoretical computer science, operations research (the science of optimization), and newest of all, bioinformatics. Although each of these subjects shares with mathematics many foundational tools, each has its own distinctive character, methodologies, standards, and accomplishments.

The mathematical science that ordinary individuals most often encounter is statistics, originally the science of the state (as in census). Statistics underlies every clinical trial, every opinion survey, and every government economic report. Yet school curricula still primarily serve to prepare students only for traditional college mathematics. School mathematics places relatively little emphasis on topics designed to build a bridge from arithmetic to the subtle and fascinating world of statistics. Recognizing this neglect, the American Statistical Association (ASA) and the NCTM have cooperated for many years in a campaign to infuse more exploratory data analysis and elementary statistics into school curricula. This effort, interestingly, is called the "Quantitative Literacy Project." (Project founders chose quantitative literacy rather than statistics as a title because they anticipated public anxiety about the term *statistics*.)

Despite its occasional use as a euphemism for statistics in school curricula, quantitative literacy is not the same as statistics. Neither is it the same as mathematics, nor is it (as some fear) watered-down mathematics. Quantitative literacy is more a habit of mind, an approach to problems that employs and enhances both statistics and mathematics. Unlike statistics, which is primarily about uncertainty, numeracy is often about the logic of certainty. Unlike mathematics, which is primarily about a Platonic realm of abstract structures, numeracy is often anchored in data derived from and attached to the empirical world. Surprisingly to some, this inextricable link to reality makes quantitative reasoning every bit as challenging and rigorous as mathematical reasoning. (Indeed, evidence from Advanced Placement examinations suggests that students of comparable ability find data-based statistical reasoning more difficult than symbol-based mathematical reasoning.)

Connecting mathematics to authentic contexts demands delicate balance. On the one hand, contextual details camouflage broad patterns that are the essence of mathematics; on the other hand, these same details offer associations that are critically important for many students' long-term learning. Few can doubt that the tradition of decontextualized mathematics instruction has failed many students, including large numbers of women and minorities, who leave high school with neither the numeracy skills nor the quantitative confidence required in contemporary society. The tradition of using mathematics as a filter for future academic performance is reinforced by increasing demand for admission to selective colleges and universities. These pressures skew school curricula in directions that are difficult to justify because they leave many students functionally innumerate.

Whereas the mathematics curriculum has historically focused on school-based knowledge, quantitative literacy involves mathematics acting in the world. Typical numeracy challenges involve real data and uncertain procedures but require primarily elementary mathematics. In contrast, typical school mathematics problems involve simplified numbers and straightforward procedures but require sophisticated abstract concepts. The test of numeracy, as of any literacy, is whether a person naturally uses appropriate skills in many different contexts.

Educators know all too well the common phenomenon of compartmentalization, when skills or ideas learned in one class are totally forgotten when they arise in a different context. This is an especially acute problem for school mathematics, in which the disconnect from meaningful contexts creates in many students a stunning absence of common number sense. To be useful for the student, numeracy needs to be learned and used in multiple contexts—in history and geography, in economics and biology, in agriculture and culinary arts (Steen, 1998, 2000). Numeracy is not just one among many subjects but an integral part of all subjects.

Elements of Quantitative Literacy

The capacity to deal effectively with the quantitative aspects of life is referred to by many different names, among them quantitative literacy, numeracy, mathematical literacy, quantitative reasoning, or sometimes just plain "mathematics." Different terms, however, convey different nuances and connotations that are not necessarily interpreted in the same way by all listeners.

An early definition of the term *numerate,* widely cited by mathematics educators, appeared in a British government report on mathematics education (Cockcroft, 1982):

> We would wish the word numerate to imply the possession of two attributes. The first of these is an "at homeness" with numbers and an ability to make use of mathematical skills which enables an individual to cope with the practical demands of everyday life. The second is an ability to

> have some appreciation and understanding of information which is presented in mathematical terms.

The same two themes emerged in the National Adult Literacy Survey (NCES, 1993), which defined *quantitative literacy* as:

> The knowledge and skills required to apply arithmetic operations, either alone or sequentially, using numbers embedded in printed material (e.g., balancing a checkbook, completing an order form).

The National Center for Education Statistics (NCES) defines the closely related knowledge and skills required to locate and use information (for example, in payroll forms, transportation schedules, maps, tables, and graphs) as *document literacy*. In contrast, the International Life Skills Survey (ILSS, 2000) currently underway defines *quantitative literacy* in a much more comprehensive manner as:

> An aggregate of skills, knowledge, beliefs, dispositions, habits of mind, communication capabilities, and problem solving skills that people need in order to engage effectively in quantitative situations arising in life and work.

The Programme for International Student Assessment (PISA, 2000) adopts a similar definition but calls it *mathematics literacy:*

> An individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgements and to engage in mathematics in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen.

From just these four definitions significant differences emerge. Some focus on an individual's ability to use quantitative tools, others on the ability to understand and appreciate the role of mathematical and quantitative methods in world affairs. Some emphasize basic skills ("arithmetic operations"), others higher-order thinking ("well-founded judgements"). To clarify these different definitions, as well as to make them more useful, we break them into different elements, which may be combined, as atoms in molecules, to form a more comprehensive portrait of quantitative literacy. Here are some of these elements:

Confidence with Mathematics. Being comfortable with quantitative ideas and at ease in applying quantitative methods. Individuals who are quantitatively confident routinely use mental estimates to quantify, interpret, and check other information. Confidence is the opposite of "math anxiety"; it makes numeracy as natural as ordinary language.

Cultural Appreciation. Understanding the nature and history of mathematics, its role in scientific inquiry and technological progress, and its importance for comprehending issues in the public realm.

Interpreting Data. Reasoning with data, reading graphs, drawing inferences, and recognizing sources of error. This perspective differs from traditional mathematics in that data (rather than formulas or relationships) are at the center.

Logical Thinking. Analyzing evidence, reasoning carefully, understanding arguments, questioning assumptions, detecting fallacies, and evaluating risks. Individuals with such habits of inquiry accept little at face value; they constantly look beneath the surface, demanding appropriate information to get at the essence of issues.

Making Decisions. Using mathematics to make decisions and solve problems in everyday life. For individuals who have acquired this habit, mathematics is not something done only in mathematics class but a powerful tool for living, as useful and ingrained as reading and speaking.

Mathematics in Context. Using mathematical tools in specific settings where the context provides meaning. Notation, problem-solving strategies, and performance standards all depend on the specific context.

Number Sense. Having accurate intuition about the meaning of numbers, confidence in estimation, and common sense in employing numbers as a measure of things.

Practical Skills. Knowing how to solve quantitative problems that a person is likely to encounter at home or at work. Individuals who possess these skills are adept at using elementary mathematics in a wide variety of common situations.

Prerequisite Knowledge. Having the ability to use a wide range of algebraic, geometric, and statistical tools that are required in many fields of postsecondary education.

Symbol Sense. Being comfortable using algebraic symbols and at ease in reading and interpreting them, and exhibiting good sense about the syntax and grammar of mathematical symbols.

These elements illuminate but do not resolve the linguistic confusions that permeate discussions of quantitative literacy. Sometimes the terms *quantitative* and *mathematical* are used interchangeably, but often they are used to signify important distinctions—for example, between what is needed for life (quantitative) and what is needed for education (mathematics), or between what is needed for general school subjects (quantitative) and what is needed for engineering and physical science (mathematics). For some the word *quantitative* seems too limiting, suggesting numbers and calculation rather than reasoning and logic, while for others the term seems too vague, suggesting a diminution of emphasis on traditional mathematics. Similarly, the term *literacy* conveys different meanings: for some it suggests a minimal capacity to read, write, and calculate, while for others it connotes the defining characteristics of an educated (literate) person.

In terms of what is needed for active and alert participation in contemporary society, quantitative literacy can be viewed as a direct analog of verbal literacy. At a fundamental level we teach the skills of reading, writing, and calculating, the principal goals of lower schools. But these basic skills are no longer sufficient to sustain a successful career or to participate fully in a modern democratic society. Today's well-educated citizens require sophistication in both literacy and numeracy to think through subtle issues that are communicated in a collage of verbal, symbolic, and graphic forms. In addition, they need the confidence to express themselves in any of these modern forms of communication. In the twenty-first century, literacy and numeracy will become inseparable qualities of an educated person.

Expressions of Quantitative Literacy

A different way to think about quantitative literacy is to look not at definitions but at actions, not at what numeracy is but at how it is expressed. Many manifestations are commonplace and obviously important, yet they are not the real reason for the increasing emphasis on numeracy.

Examples:

- Estimating how to split a lunch bill three ways
- Comparing price options for leasing or purchasing a car
- Reading and understanding nutrition labels
- Reconciling a bank statement and locating the sources of error
- Scaling recipes up and down and converting units of volume and weight
- Mentally estimating discounts, tips, and sales prices
- Understanding the effects of compound interest
- Reading bus schedules and maps

More relevant to current students and future citizens are many of the more sophisticated expressions of quantitative reasoning that have become common in our data-driven society. Some of these serve primarily personal ends, while others serve the goals of a democratic society. Together they provide a rich portrait of numeracy in the modern world.

Citizenship

Virtually every major public issue—from health care to social security, from international economics to welfare reform—depends on data, projections, inferences, and the kind of systematic thinking that is at the heart of quantitative literacy. Examples:

- Understanding how resampling and statistical estimates can improve the accuracy of a census
- Understanding how different voting procedures (e.g., runoff, approval, plurality, preferential) can influence the results of elections
- Understanding comparative magnitudes of risk and the significance of very small numbers (e.g., 10 ppm or 250 ppb)
- Understanding that unusual events (such as cancer clusters) can easily occur by chance alone
- Analyzing economic and demographic data to support or oppose policy proposals
- Understanding the difference between rates and changes in rates, for example, a decline in prices compared with a decline in the rate of growth of prices

- Understanding the behavior of weighted averages used in ranking colleges, cities, products, investments, and sports teams
- Appreciating common sources of bias in surveys such as poor wording of questions, volunteer response, and socially desirable answers
- Understanding how small samples can accurately predict public opinion, how sampling errors can limit reliability, and how sampling bias can influence results
- Recognizing how apparent bias in hiring or promotion may be an artifact of how data are aggregated
- Understanding quantitative arguments made in voter information pamphlets (e.g., about school budgets or tax proposals)
- Understanding student test results given in percentages and percentiles and interpreting what these data mean with respect to the quality of schools

Culture

As educated men and women are expected to know something of history, literature, and art, so should they know—at least in general terms—something of the history, nature, and role of mathematics in human culture. This aspect of quantitative literacy is most commonly articulated in goals colleges set forth for liberal education. Examples:

- Understanding that mathematics is a deductive discipline in which conclusions are true only if assumptions are satisfied
- Understanding the role mathematics played in the scientific revolution and the roles it continues to play
- Understanding the difference between deductive, scientific, and statistical inference
- Recognizing the power (and danger) of numbers in shaping policy in contemporary society
- Understanding the historical significance of zero and place value in our number system
- Knowing how the history of mathematics relates to the development of culture and society
- Understanding how assumptions influence the behavior of mathematical models and how to use models to make decisions

Education

Fields such as physics, economics, and engineering have always required a strong preparation in calculus. Today, other aspects of quantitative literacy (e.g., statistics and discrete mathematics) are also important in these fields. Increasingly, however, other academic disciplines are requiring that students have significant quantitative preparation. Examples:

- Biology requires computer mathematics (for mapping genomes), statistics (for assessing laboratory experiments), probability (for studying heredity), and calculus (for determining rates of change).
- Medicine requires subtle understanding of statistics (to assess clinical trials), of chance (to compare risks), and of calculus (to understand the body's electrical, biochemical, and cardiovascular systems).
- The social sciences rely increasingly on data either from surveys and censuses or from historical or archeological records; thus statistics is as important for a social science student as calculus is for an engineering student.
- Advances in scientific understanding of the brain have transformed psychology into a biological science requiring broad understanding of statistics, computer science, and other aspects of quantitative literacy.
- The stunning impact of computer graphics in the visual arts (film, photography, sculpture) has made parts of mathematics, especially calculus, geometry, and computer algorithms, very important in a field that formerly was relatively unquantitative.
- Interpretation of historical events increasingly depends on analysis of evidence provided either by numerical data (e.g., government statistics, economic indicators) or through verification and dating of artifacts.
- Even the study of language has been influenced by quantitative and logical methods, especially in linguistics, concordances, and the new field of computer translation.

Professions

As interpretation of evidence has become increasingly important in decisions that affect people's lives, professionals in virtually every field are now expected to be well versed in quantitative tools. Examples:

- Lawyers rely on careful logic to build their cases and on subtle arguments about probability to establish or refute “reasonable doubt.”
- Doctors need both understanding of statistical evidence and the ability to explain risks with sufficient clarity to ensure “informed consent.”
- Social workers need to understand complex state and federal regulations about income and expenses to explain and verify their clients’ personal budgets.
- School administrators deal regularly with complex issues of scheduling, budgeting, inventory, and planning—all of which have many quantitative dimensions.
- Journalists need a sophisticated understanding of quantitative issues (especially of risks, rates, samples, surveys, and statistical evidence) to develop an informed and skeptical understanding of events in the news.
- Chefs use quantitative tools to plan schedules, balance costs against value of ingredients, and monitor nutritional balance of meals.
- Architects use geometry and computer graphics to design structures, statistics and probability to model usage, and calculus to understand engineering principles.

Personal Finance

Managing money well is probably the most common context in which ordinary people are faced with sophisticated quantitative issues. It is also an area greatly neglected in the traditional academic track of the mathematics curriculum. Examples:

- Understanding depreciation and its effect on the purchase of cars or computer equipment
- Comparing credit card offers with different interest rates for different periods of time
- Understanding the relation of risk to return in retirement investments
- Understanding the investment benefits of diversification and income averaging
- Calculating income tax and understanding the tax implications of financial decisions
- Estimating the long-term costs of making lower monthly credit card payments

- Understanding interactions among different factors affecting a mortgage (e.g., principal, points, fixed or variable interest, monthly payment, and duration)
- Using the Internet to make decisions about travel plans (routes, reservations)
- Understanding that there are no schemes for winning lotteries
- Choosing insurance plans, retirement plans, or finance plans for buying a house

Personal Health

As patients have become partners with doctors in making decisions about health care and as medical services have become more expensive, quantitative skills have become increasingly necessary in this important aspect of people's lives. Examples:

- Interpreting medical statistics and formulating relevant questions about different options for treatment in relation to known risks and the specifics of a person's condition
- Understanding medical dosages in relation to body weight, timing of medication, and drug interactions
- Weighing costs, benefits, and health risks of heavily advertised new drugs
- Understanding terms and conditions of different health insurance policies; verifying accuracy of bills and insurance payments
- Calibrating eating and exercise habits in relation to health
- Understanding the impact of outliers on summaries of medical data

Management

Many people need quantitative skills to manage small businesses or non-profit organizations as well as to fulfill their responsibilities when they serve on boards or committees that are engaged in running any kind of enterprise. Examples:

- Looking for patterns in data to identify trends in costs, sales, and demand
- Developing a business plan, including pricing, inventory, and staffing strategies for a small retail store

- Determining the break-even point for manufacturing and sale of a new product
- Gathering and analyzing data to improve profits
- Reviewing the budget of a small nonprofit organization and understanding relevant trends
- Understanding the limitations of extrapolating from data in a fixed range
- Calculating time differences and currency exchanges in different countries

Work

Virtually everyone uses quantitative tools in some way in relation to their work, if only to calculate their wages and benefits. Many examples of numeracy on the job are very specific to the particular work environment, but some are not. Examples:

- Producing a schedule or tree diagram for a complicated project
- Researching, interpreting, and using work-related formulas
- Using spreadsheets to model different scenarios for product sales and preparing graphs that illustrate these options
- Understanding and using exponential notation and logarithmic scales of measurement
- Maintaining and using quality control charts
- Optimizing networks to develop efficient ways to plan work processes
- Understanding the value of statistical quality control and statistical process control

Skills of Quantitative Literacy

For a different and more traditional perspective on quantitative literacy, we might create an inventory of quantitative skills expected of an educated person in contemporary society. For many, a list of skills is more comforting than a list of elements or expressions because skills are more immediately recognizable as something taught and learned in school. Moreover, many people believe that skills must precede applications and that once learned, quantitative skills can be applied whenever needed. Unfortunately, considerable evidence about the associative nature of learning suggests that this approach works very imperfectly. For most students, skills learned free of context are skills devoid of meaning and utility. To be effective, numeracy skills must be taught and learned in settings that are both meaningful and memorable.

Nevertheless, a list of skills is a valuable enhancement to our emerging definition of quantitative literacy—a third dimension, so to speak, which complements the foregoing analyses in terms of elements and expressions. A list of skills helps instructors plan curricula to cover important topics and helps examiners assess the desired balance of knowledge. An appendix to the Mathematical Association of America's report on quantitative literacy (Sons, 1996) offers—with suitable apologies and caveats—a consensus among mathematicians on skills that are especially important for courses in quantitative literacy. This list includes predictable topics from arithmetic, geometry, and algebra that are part of every school mathematics program, but it also includes many newer topics from statistics and optimization that are usually offered to students, if at all, only as electives.

In fact, many of these "elective" skills are firmly embedded in the elements and expressions of quantitative literacy. They include:

- *Arithmetic:* Having facility with simple mental arithmetic; estimating arithmetic calculations; reasoning with proportions; counting by indirection (combinatorics).
- *Data:* Using information conveyed as data, graphs, and charts; drawing inferences from data; recognizing disaggregation as a factor in interpreting data.
- *Computers:* Using spreadsheets, recording data, performing calculations, creating graphic displays, extrapolating, fitting lines or curves to data.
- *Modeling:* Formulating problems, seeking patterns, and drawing conclusions; recognizing interactions in complex systems; understanding linear, exponential, multivariate, and simulation models; understanding the impact of different rates of growth.
- *Statistics:* Understanding the importance of variability; recognizing the differences between correlation and causation, between random-

ized experiments and observational studies, between finding no effect and finding no statistically significant effect (especially with small samples), and between statistical significance and practical importance (especially with large samples).

- *Chance:* Recognizing that seemingly improbable coincidences are not uncommon; evaluating risks from available evidence; understanding the value of random samples.
- *Reasoning:* Using logical thinking; recognizing levels of rigor in methods of inference; checking hypotheses; exercising caution in making generalizations.

The differences between these topics and those found on many tests or in courses designed to meet a so-called mathematics or quantitative requirement are typical of the distinction between quantitative literacy, which stresses the use of mathematical and logical tools to solve common problems, and what we might call *mathematical literacy*, which stresses the traditional tools and vocabulary of mathematics. Indeed, it is not uncommon for a person who is familiar with a mathematical or statistical tool (e.g., the formula for standard deviation) not to recognize in a real-life situation when it should be used—or just as important, when it should not be used. Similarly, it is not uncommon for someone who knows how to use standard deviation in a specific quality control setting not to recognize the concept when it arises in a different context (such as in a course in economics).

Quantitative Literacy in Context

In contrast to mathematics, statistics, and most other school subjects, quantitative literacy is inseparable from its context. In this respect it is more like writing than like algebra, more like speaking than like history. Numeracy has no special content of its own, but inherits its content from its context.

Another contrast with mathematics, statistics, and most sciences is that numeracy grows more horizontally than vertically. Mathematics climbs the ladder of abstraction to see, from sufficient height, common patterns in seemingly different things. Abstraction is what gives mathematics its power; it is what enables methods derived in one context to be applied in others. But abstraction is not the focus of numeracy. Instead, numeracy clings to specifics, marshaling all relevant aspects of setting and context to reach conclusions.

To enable students to become numerate, teachers must encourage them to see and use mathematics in everything they do. Numeracy is driven by issues that are important to people in their lives and work, not by future needs of the few who may make professional use of mathematics or statistics. In teaching quantitative literacy, content is inseparable from pedagogy and context is inseparable from content. Fortunately, because numeracy is ubiquitous, opportunities abound to teach it throughout the curriculum. Only by encountering the elements and expressions of numeracy in real contexts that are meaningful to them will students develop the habits of mind of a numerate citizen. Like literacy, numeracy is everyone's responsibility.

Challenges of Quantitative Literacy

The penetration of numeracy into all aspects of life—from education, work, and health to citizenship and personal finance—confronts us with a rapidly evolving phenomenon that we understand at best imperfectly. Americans have had decades, even centuries, to recognize the public importance of literacy. Campaigns for literacy are commonplace, now even part of presidential politics. Yet there is little corresponding public concern about numeracy, except for ill-informed (and innumerate) obsession about SAT scores and AP calculus enrollments. The public seems not to grasp either the escalating demands for quantitative literacy or the consequences of widespread innumeracy.

Ironically, public apathy in the face of innumeracy may itself be a consequence of innumeracy. People who have never experienced the power of quantitative thinking often underestimate its importance, especially for tomorrow's society. In contrast, because it has been a staple of the school curriculum, most adults do recognize the importance of mathematics even if they themselves do not feel comfortable with it and have a highly distorted impression of its true nature. But as we have seen, numeracy is not mathematics, and public concern about mathematics education does not automatically translate into a demand for quantitative literacy.

Thus a key challenge in the campaign for quantitative literacy is to mobilize various constituencies for whom numeracy is especially important. The quality of medical care, for example, depends on numerate patients, just as wise public policy depends on numerate citizens. Educational, business, and political leaders all have a stake in a numerate public (even if they sometimes rely on the public's innumeracy to promote questionable products or policies). These leaders, however, naturally focus their attention on existing instruments such as mathematics standards, high school graduation tests, college admission tests, college placement tests, and (occasionally) college graduation requirements.

If, as seems inescapable, the importance of quantitative literacy will become ever more apparent and pressing (albeit in different ways to different groups), a second challenge is to expand these traditional instruments of educational policy to include stronger emphasis on quantitative literacy. Indeed, as the twenty-first century unfolds, quantitative literacy will come to be seen not just as a minor variation in the way we functioned in the twentieth century but as a radically transformative vantage point from which to view education, policy, and work.

THE DESIGN TEAM

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Like any committee effort, this case statement represents not unanimity of views but a consensus on important issues that members of the Design Team believe are both timely and urgent.

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