P(h|e)
P(e|h)
P(e|~h)

PROBABILITY: CLASSICAL AND BAYESIAN

Colloquium
University of Northern Iowa
December 14, 1998

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Probability Classical and Bayesian

Statisticians are

- ***united*** **on the axioms of statistics (mathematics)**
- ***divided*** **on the meaning of chance (philosophy)**

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United on Probability Axioms

1. P(a) ≥ 0 for all a in domain of P
2. P(t) = 1 if t is a tautology
3. P(a ∨ b) = P(a) + P(b)
   if a, b and a∨b are all in domain of P
   and if a and b are mutually exclusive
4. P(h|e) = P(h & e)/P(e)

P(h|e)
P(e|h)
P(e|~h)

United on Bayes Theorems

Bayes version:

P(h|e) = P(e|h) P(h)/P(e)

LaPlace version:

P(h|e)= P(h)/[P(h)+P(~h) LR]

LR = Likelihood Ratio = P(e|~h)/P(e|h)

P(e) = P(e|h)P(h) + P(e|~h)P(~h)

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Probability: Classical versus Bayesian

Classical probability is objective:

- expresses fundamental laws regarding the assignment of objective physical probabilities to events in the outcome space of stochastic experiments
- independent of our feelings
- a property of the future: not of the past

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Probability: Classical versus Bayesian

Bayesian probability is epistemic -- based on our context of knowledge

- expresses numeric degrees of uncertainty
- measures our strength of belief
- can be applied to the truth of propositions

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Probability: Classical versus Bayesian

- ***Classical (Purely objective)***
  Hypothesis testing with p-values
  ***Confidence* that fixed parameter is in a range**
- ***Bayesian strength of belief***
  No hypothesis testing; no p-values
  ***Probability* fixed parameter is in fixed range**

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Teaching Bayesian: Yes! Realistic approach

"…differences of opinion are the norm in science and an approach [Bayesian] that explicitly recognizes such differences is realistic." [*Statistics: A Bayesian Perspective* by Berry]

"The Bayesian approach is the only one capable of representing faithfully the basic principles of scientific reasoning."
[*Scientific Reasoning* by Howson and Urbach]

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Teaching Bayesian: No! "at best, premature"

"*Surveys of the statistical methods actually in use suggest that Bayesian techniques are little used.*

Bayesians have not yet agreed on standard approaches to standard problems settings.

Bayesian reasoning requires a grasp of conditional probability, a concept confusing to beginners.

Finally, an emphasis on Bayesian inference might well impede the trend toward experience with real data…"

David Moore, 1997

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"Bayesian Interpretation of Classical Hypothesis Tests"

- **Combines classical hypothesis test with Bayesian strength of belief.**
- **If prior belief about truth of null is 50%, P(alternate is false|reject null) = p-value**
- **Objectively determines prior strength of belief necessary to achieve a 95% probability that the alternate is true.**

Milo Schield, 1995 ASA JSM

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"Bayesian Interpretation of Classical Confidence"

Interpretes classical confidence as a Bayesian strength of belief.

One should be indifferent in betting on
- **whether next ball is red (given 95% chance)**
- **whether a particular 95% confidence interval contains the population parameter**

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Conclusion

- ***Students take statistics to help them make better decisions.***
- ***Decision making is Bayesian -- based on a strength of belief.***
- ***Elementary statistics should include a Bayesian interpretation of classical statistical inference.***

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"Statistical Literacy and Evidential Statistics"

- **Focus on observational studies**
- **Focus on confounding factors**
- **Emphasize conditional probability**
- **Clearly identify role of chance:**
  - **Highly unlikely if due to chance"**
  - **highly unlikely to be due to chance"**

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"Statistical Literacy and Simpson's Paradox"

- **Simpson's Paradox: a reversal of an association due to a confounding factor.**
- **Objectively determines the minimum effect size for a reversal in the three variable case.**

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Elementary Statistics: Technical versus Basic

Elementary Statistics should be split:

- ***Technical statistics*** **for majors that use hypothesis tests (psychology, sociology, education, etc.)**
- ***Basic statistics*** **for majors that don't (humanities) and students that don't (two-year schools)**

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Elementary Statistics: Technical versus Basic

- ***Technical Statistics*****:**
  Statistical inference: sampling distributions, confidence intervals and hypothesis tests
- ***Basic Statistics*****:**
  Reading tables, reading and interpreting graphs, and evaluating the results of observational studies.

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Elementary Statistics: Benefits of Changes

- ***Goal is statistical literacy: critical thinking about statistics***
- ***Opportunity to Improve:***
  Statistical education
  Reputation of statistics
- ***Attract national attention***
  Demonstrate leadership

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Elementary Statistics: (To be continued)

Need more research on

- **assessment of statistical literacy**
- **student comprehension/retention**
- **selection of topics**
- **development of teaching materials**
- **value added for other majors**
- **difficulty of training faculty**

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Statistics Faculty Bayesian: US and UK

- *US & Canada:*
  - *0 - 10% Pure Bayesian***
  - *10 - 30% Mixed Bayesian***
- *UK, Australia, & New Zealand:*
  - *20 - 40% Pure Bayesian***
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  - *** Estimated*

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Math program enrollments Two-year colleges

Enrollment in elementary statistics

- *11,000 in 1970*
- *20,000 in 1980 -- 6.0% growth/year*
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77% of all enrollment in elementary statistics is at the 4-year level

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Math program enrollments: Four-year colleges

Enrollment: 1995 versus 1990

- *25% increase in elementary stats*
- *10% decrease in math courses*
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Why are more students taking stats?

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