PHILOSOPHICAL ISSUES IN STATISTICAL INFERENCE

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Abstract: In essence, statistics is a part of philosophy. Specifically, statistics and logic both are part of epistemology. This paper examines three major issues in statistics: the role of statistics in science (philosophy of science), the role of objectivity in defining probability, and the role of chance in statistical inference. In each issue there are two sides: classical frequency-based objectivists and Bayesian strength of belief subjectivists. The ongoing debate within statistics is primarily philosophical. The root issue is the problem of induction – also known as the problem of universals. The problem of induction is one of the great problems in philosophy today. The ongoing conflict in statistics is in large part a reflection of the failure of philosophers to solve this classic problem.

This paper examines four topics: the relation between statistics and philosophy of science, the nature of probability, the role of chance in inference, and hypothesis testing. Specific recommendations are offered.

INTRODUCTION

Although statistics is seldomly taught by philosophers, statistics is as close to philosophy as is logic. Only the emphasis on quantity places it in the purview of mathematicians. This paper examines the relation between statistics and philosophy in four key areas.

1. AMBIGUITIES IN STATISTICAL CLAIMS

The reason people can 'lie' with statistics is that many of the key phrases used in statistics are ambiguous. These ambiguities are exploited by the knowledgable, communicated by those who could know better and accepted by those who trust those who should know more.

A keyword is 'can'. 'Can' is ambiguous.
1. This drug can kill -- anyone at a high enough dose.
2. This drug can kill – some subjects but not all.
3. This drug can kill. Although I don't know any circumstances under which it definitely will kill all or some, I do have some reasons to believe it could.
4. This drug can kill – because there is no reason it can't (although there is no reason to think it can: #3).

These four forms of 'can' represent different kinds of potentiality. The first two identify the conditions under which the potency will be actualized. The third indicates one's strength of belief that the potency exists although there is no certainty that it can (#1 or #2). The fourth is a statement of conceivability: 'can' meaning it is conceivable (not logically impossible).

Statistical ambiguity is present in the three key forms of statistical inference: generalization, prediction and explanation.

Generalization: *Statistics will generalize from a particular to a universal and identify the confidence associated with the inference.* This seems incredible until one realizes that the generalization accounts for only one kind of uncertainty – that due entirely to chance. A statistical generalization cannot estimate the effect of any other kind of uncertainty.

Prediction: Statistics "predicts" the future from past observations. But this is no great feat provided one has the implicit assumption of ceterus paribus about everything related to the model that has been developed.

Explanation: *Statistics can explain which things are factors or influences. Statistics identifies how strongly a given factor "explains" a particular relationship.* This too seems incredible until one realizes that this explanation is merely a way of saying that two things are related, associated or correlated. The measurement of this association is therefore non-controversial. The problem is when unwary readers read that "A influences B", "A is a significant factor in the generation of B", or "A explains 60% of the changes in B". Most readers assume these are statements about causality: whether A causes B and how strongly a change in A relates to a change in B.

Association, prediction and explanation each have two forms (observational and experimental) that are easily confused. Consider data obtained on the prices for a group of houses.

Observational: An observational association says that for an 'increase' of one bathroom, the mean value of the houses increased by $25,000. An observational prediction says that for an 'increase' of one bathroom, the expected value of the house will increase by $25,000. An observational explanation says that an 'increase' of one bathroom explains the expected increase of $25,000 in the value of the house. In each case, the 'increase' was due to a shift in focus from houses having one bath to houses having two baths, or from two to three, etc. In an observational association, prediction or explanation, the only 'change' is in our choice of subjects being observed. The 'change' is entirely mental – not physical.

Experimental: An experimental association says that for an 'increase' of each bathroom (physically adding another bathroom), the mean value of such houses increased by $10,000. An experimental prediction says that for an 'increase' of one bathroom (physically adding another bathroom), the expected value of that house will increase by $10,000. An experimental explanation says that adding one bathroom to an existing house 'explains' the expected increase of $10,000. In each case, the 'increase' was physical – an internal change within the given subjects. The increase was a physical change in reality. – not just a shift in mental focus.

Obviously an observational statement is based on an association, whereas an experimental statement is based on causality.

This obfuscation between observational and experimental is quite common in arguments involving statistics. The truth of an observational association is used to argue for the truth of an experimental explanation or an experimental prediction.

1. Cars with cell phones have a 50% higher rate of accidents than cars without. Thus to minimize accidents we should ban car phones.

2. People who complete college make $6,000 more per year than people who do not. Thus if we want people to have better incomes, we should encourage them to complete college.

3. People who take vitamins have half as many medical problems as those who do not. If we want to improve health, we should subsidize the purchase of vitamins.

2. NATURE OF PROBABILITY

Typically, probability is presented in three forms:

1. Hypothetical: "Suppose we have a fair coin…."
2. Empirical: In the long run, the proportion of heads in flipping a fair coin will approach 50%.
3. Subjective: I think the probability of electing a Libertarian President will continue to increase.

The first two are often grouped together as being objective. The last one (subjective) uses probability to indicate one's strength of belief in the truth of a claim. The claim may be historical ("I think the Dinosaurs were killed by a meteor.") or it may be a unique future event ("I think that Gold will become the currency of choice in less than 50 years.").

Classical statisticians do not use chance or probability to describe a fact. Suppose someone won two lotteries in one week. Instead of saying the chance of this happening is 1 in a million, they would say, "the chance of this [kind of thing] happening is one in a million" or else they would say, "the chance of this is obviously 1 – it happened."

Classical statisticians use the word 'confidence' to describe a fact whose value is unknown. Now suppose one has bough two lottery tickets. The winning numbers have been drawn, but the ticket holder is ignorant of the winning numbers. Classical statisticians would say one should be 0.000001% confident of wining both lotteries. See Schield, 1997 for a discussion of the relation between confidence and probability.

Objective and subjective probabilities both obey the same probability calculus (laws of probability) despite their differences concerning the nature of probability,. They both have the same algebraic rules for exclusivity, independence and Bayesian dependence.

Exclusivity: If A and B are exclusive, then having both exist or be true simultaneously is impossible. If 30% of the students are sophomores and 20% are seniors and one cannot be both, then the probability of being either must be 50%. Similarly, if the "probability" that A committed a crime is 10% and the "probability" that B committed the crime is 20%, then the "probability" that one of them committed the crime must be 30%.

Independence: If A and B are independent, then the existence of one has no effect on the chance of the other existing or being true. If 50% of the students are male and if 30% are business majors and if being a business major is independent of sex, then we must conclude that 15% of all students are male business majors. Similarly if we think there is a 20% "probability" of a stock market drop of at least 30 percentage points and if we think there is a 10% probability of a major war, and if these events are independent, then the probability of both must be exactly 3%.

Bayesian Dependence: Bayes Rule is a most interesting example of reasoning from consequent to antecedent. Suppose that subjects are either diseased or disease-free. Suppose that test results on these subjects are either positive or negative.

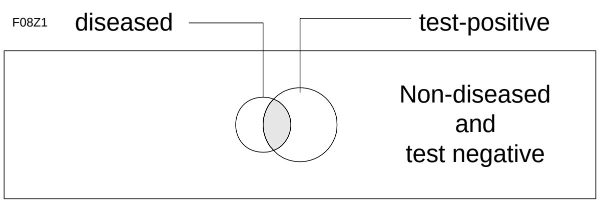
1. If diseased, then 90% chance of positive test.
2. If not-diseased, then 90% chance of negative test.
3. Suppose there is a 10% chance of being diseased.

Then the chance of a subject being diseased is

a. 50% given a positive test, and
b. 1% given a negative test.

The quality of a test (90% sensitivity and 90% specificity) and the prevalence of the disease (10%) logically determine the quality of the prediction (50% if positive and 1% if negative).

This is a simple application of conditional probability as summarized by this Venn diagram.



The underlying issue between Bayesians and non-Bayesians is objectivity. The classical statisticians object to allowing subjective probabilities. To do so would undermine the objectivity of science and actually retard the attainment of knowledge. Bayesians hold that probability is inherently subjective and that the classical statisticians are dealing with Platonic forms as being intrinsic and are thus non-objective. Bayesians hold that subjective probabilities are informed by experience. As more experience accumulates, it will dilute out the differences between different prior probabilities. Bayesians hold that this is how knowledge is accumulated and that using classical tests actually distorts and subverts the acquisition of meaningful knowledge.

For a good overview of both sides, see Howson and Urbach (1993) on the Bayesian side and Mayo (1996) on the classical side.

3. CHANCE IN INFERENCE

There is verbal obfuscation about whether chance is to be treated as a premise (as given) or as a conclusion (an explanation) whose truth is disputable and needs support from an argument. The following questions were posted on the SciStatEdu newsgroup:

1. Suppose I flip a coin and get 9 heads in 10 tries. Statisticians would say this outcome is highly unlikely (a) IF due to chance, (b) TO BE due to chance or (c) DUE to chance." [I'd pick 'a'. Students pick 'b' or 'c'. I think 'c' is ambiguous.]

2. If an outcome is statistically significant, this is the same as saying it is highly unlikely (a) IF due to chance, (b) TO BE due to chance or (c) DUE to chance." I'd pick 'a'. Students pick 'b' or 'c'. I think 'c' is ambiguous.]

3. Given that an association (or outcome) is statistically significant, _______ it is unlikely to be due to chance. (a) THIS MEANS {Same meaning; different words} (b) this PROVES it is unlikely to be due to chance. {Logically necessary} or (c) this is EVIDENCE that it is unlikely to be due to chance. [I'd pick 'c'; I think students would pick either 'a' or 'b']

4. Given that an association (or outcome) is unlikely to be due to chance, this _______ it is likely to be due to a determinate cause. Choose from (a) MEANS, (b) PROVES or (c) IS EVIDENCE. [I'd pick either 'a' or 'b' -- but I'm not very certain yet...]

Replies varied. One said this was verbal obfuscation that had no merit and would simply turn students off. Another said these distinctions were among the most fundamental concepts in all of statistics.

4. HYPOTHESIS TESTING

Schield, 1996, argued that classical hypothesis testing is vulnerable to Type 1 error provided the alternate hypothesis is extremely unlikely prior to sampling. This is the prosecutor's fallacy.

5. CHANCE AND PHILOSOPHY

There are two views of chance: epistemological and metaphysical. In the epistemic view, chance is merely the name for those causes that are so numerous and so indeterminate that collectively they are called 'chance.' In the metaphysical view, chance is something in reality whose existence remains independent of the quality of our knowledge or measurements.

6. PHILOSOPHY OF SCIENCE & STATISTICS

Statistics is the key to the philosophy of science today. This was not the case 100 years ago. At that time, causality in the natural world was typically determinate. Probabilistic causality and the influence of chance were reserved for games of chance – not philosophy of science. Today, probabilistic causality is much more common – not only in medicine and health but also in quantum physics, statistical thermodynamics, and astrophysics. It has always been common in predicting weather and throughout the social sciences.

6. CONCLUSION

Statistics is in dire need of philosophical direction. One professor at the Univ. of Minnesota asserted that at least 30% of all Ph.D. theses in Psychology could not be replicated – although all had passed a statistical test of significance. An article in Science (1997) made the same point. Statistics needs help from Philosophy in setting its reasoning in order.

REFERENCES

Howson, Colin and Peter Urbach (1993). *Scientific Reasoning* 2nd Ed. Open Court Publishing.

Kelley, David (1994). *The Art of Reasoning*. 2nd Ed.

Lee, Peter (1989). *Bayesian Statistics, An Introduction.* Edward Arnold

Mayo, Deborah (1996). *Error and the Growth of Experimental Knowledge*, The University of Chicago Press.

Moore, David & George McCabe (1993). *Introduction to the Practice of Statistics*, 2nd ed., Freeman.

Schield, Milo (1996). *Using Bayesian Reasoning in Classical Hypothesis Testing*, American Statistical Association, 1996 Proceedings of the Section on Statistical Education.

Schield, Milo (1997). *Interpreting Confidence*, American Statistical Association, 1996 Proceedings of the Section on Statistical Education.

Utts, Jessica (1991), *Seeing Through Statistics*, Duxbury Press.

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INTERPRETING

CONFIDENCE INTERVALS

CLASSICAL, BAYESIAN AND NEOCLASSICAL

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CLASSICAL CONFIDENCE INTERVALS

STRENGTHS:

1. **Relative frequency basis for probability and for confidence level.**
2. **Does not treat fixed population parameters as being variable.**

WEAKNESSES:

1. **Failure to identify at what time 'confidence' applies.**
   - Prior to sampling (as does probability).
   - After sampling but <u>before</u> the sample statistic is known.
   - After sampling and <u>after</u> the sample statistic is known.

2. **Failure to distinguish between definite and indefinite**

   Algebraically: $[\bar{x} - Z(\sigma/\sqrt{n})] \le \mu \le [\bar{x} + Z(\sigma/\sqrt{n})]$
   - Definite: $[\bar{x}_o - Z(\sigma/\sqrt{n})] \le \mu \le [\bar{x}_o + Z(\sigma/\sqrt{n})]$. $\bar{x}_o$ is fixed.
   - Indefinite: $[\tilde{\bar{x}} - Z(\sigma/\sqrt{n})] \le \mu \le [\tilde{\bar{x}} + Z(\sigma/\sqrt{n})]$. $\tilde{\bar{x}}$ is variable

   Verbally: "this interval":
   - Definite: "this particular interval" (fixed after sampling)
   - Indefinite: "this kind of interval" (random prior to sampling)

3. **Failure to identify what kind of thing confidence is.**
   - A new name for a virtual or hypothetical probability.
   - A new name for a kind of relative frequency.
   - The strength of one's belief in the truth of a claim.

More Detail on the Concept of "CONFIDENCE"

Q. What is confidence?

A. It is not a probability.

Q. What is the value of confidence?

A. Value is determined by classical relative frequency (% of intervals)

PROBLEM IN UNDERSTANDING CONCEPT OF CONFIDENCE

We now know what it is not – but not what it is

Knowing the numerical value doesn't tell us what it is.

We need to know what confidence is in positive terms (not negative).

SUMMARY:

Most presentations of confidence intervals fail to answer these three questions:

1. What kind of thing confidence is?
2. What does confidence apply to?
3. When does confidence apply?

Thus,

Students can calculate confidence intervals.

Students still can't interpret what they mean.

NEOCLASSICAL INTERPRETATION OF CONFIDENCE INTERVALS:

1. Confidence is psychological (being confident is psychological).
2. '90% confidence' (being 90% confident) identifies a level of confidence when facing a situation involving a 90% chance (probability) of winning.
3. Thus, 90% confidence is really a calibration or prescriptive of one's subjective confidence using an objective measure of uncertainty.
4. A given level of confidence can (and in some case should) persist even when the extrinsic uncertainty becomes a fact – provided one has no additional knowledge about the value of the parameter in question.

To get from probability to confidence involves three steps.

Step 1 uses the Principal Principle.

PRINCIPAL PRINCIPLE

Howson and Urbach (1993), Page 240

"The principle states that if the objective, physical probability of a random event (in the sense of its limiting relative frequency in an infinite sequence of trials) were known to be r, and if no other relevant information were available, then the appropriate subjective degree of belief that the event will occur on any particular trial would also be r."

[underscore added]

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CONFIDENCE! IN WHAT?

1. **the chance that the next random confidence interval includes μ**
2. **the chance that μ is in <u>this</u> particular confidence interval**
3. **the certainty one has in the truth of a claim (strength of belief)**

CONFIDENCE REFERS TO

STRENGTH OF BELIEF

1. **If confidence refers to the uncertainty in the next random sample, then confidence is no different than probability.**
2. **If confidence refers to the uncertainty in the value of the population parameter, then confidence is meaningless.**

   There is no uncertainty in the population parameter.
3. **By elimination, confidence must refer to one's strength of belief.**

STRENGTH OF BELIEF DOES NOT IMPLY SUBJECTIVITY

NEOCLASSICAL INTERPRETATION

OF CONFIDENCE

Consider a fair coin.

Before flipping:

1. **There is a 50% chance (probability)of getting heads**
2. **I am 50% <u>confident</u> of getting a head.**
3. **I am 100% <u>confident</u> that <u>the chance of getting a head is 50%</u>.**

After flipping but before seeing.”**:**

4. **I am 50% confident this coin is flipped with heads up.**

Consider a normal population with μ = 100 and σ = 10.
Consider random samples of size 100.

1. **A 68% probability interval for $\bar{x}$ ranges from 99 to 101.**
2. **68% of the 68% confidence intervals will include μ = 100**
3. **There is a 68% chance a random 68% CI will include μ.**
4. **I am 68% <u>confident</u> that μ is in the next 68% CI.**

5. **Given that $\bar{x}_o$ = 99, I am 68% <u>confident</u> that 98 ≤ μ ≤ 100.**

GENERAL RECOMMENDATIONS FOR TEACHING CONFIDENCE INTERVALS

1. *Use probability and confidence in a consistent fashion.*

2. *Present both the classical and neoclassical interpretations of confidence intervals*.

3. *Specify the context (the time):*
   - prior to taking a sample?
   - after taking a sample but prior to taking the statistics?
   - after taking a sample and after taking the statistics?

4. Qualify algebraic statements about confidence intervals so students can always tell what is variable.
   - If $\bar{x}$ designates a variable and unknown, then use $\tilde{\bar{x}}$.
   - If $\bar{x}$ designates a constant that is fixed, then use $\bar{x}_o$.

5. Avoid verbal ambiguity ('this interval')
   - If a particular interval say 'this particular interval'
   - If the interval is variable say, 'this kind of interval'.

6. Say what 'confidence' is rather what it is not.

NEOCLASSICAL RECOMMENDATIONS FOR TEACHING CONFIDENCE INTERVALS

7. Say what 'confidence' is rather than what it is not. Confidence is a psychological concept that measures one's strength of belief in the truth of a claim. It is typically subjective but can be prescribed objectively.

8. Show how frequentist probability can be used to calibrate one's level of confidence. Identify and validate the Principal Principle. Show that confidence is not always subjective. [Primacy of Existence]

9. Show how a given level of confidence would justify taking on a bet having a similar numerical chance (probability) of success (assuming similar loss/gain).

10. Make statements that include 'confident': "We can be 95% confident (break even on a 95% bet) that this 95% confidence interval includes the population parameter."

11. Show that the Principal Principle does not always justify inferring the existence of a particular physical uncertainty from a level of confidence. [Fallacy of Primacy of Consciousness]

Probability of a future event

CONTEXT: Urn contains 100 balls. 50% are red.

ACTION: Selecting 10 balls (with replacement) from an urn.

CLAIM: Between 40% and 60% of these balls will be red.

PROBABILITY 90% chance of this claim being true.

STRENGTH OF BELIEF:

Full statement: 90% confident that this claim is true.

Abbreviated statement: 90% confident that 40% to 60% of the next 10 balls will be red

Probability that a claim is true

CONTEXT: Urn contains 10 balls. An unknown faction are red.

ACTION: Selected 10 balls (with replacement) from urn. Five were red.

CLAIM: Percentage of red balls in the urn is between 40% and 60%

PROBABILITY Either 0% or 100% chance of this claim being true.

STRENGTH OF BELIEF:

Complete statement: 90% confident that this claim is true.

Abbreviated statement: 90% confident that true percentage is between 40% and 60%.

I'm probably not very confident.

I'm 40% confident that the probability of drawing a red ball is 50%

1. CONFIDENCE IS A KEY WORD
2. CONFIDENT IS NOT.
3. BEING CONFIDENT IS WHAT LINKS CLASSICAL AND BAYESIAN
4. CONFIDENCE IS NOT JUST ANOTHER WAY OF EXPRESSING A PROBABILITY.
5.

CLASSICAL, BAYESIAN AND NEOCLASSICAL INTERPRETATIONS OF UNCERTIANTY

Uncertainty	*Source of uncertainty (what)*	
	physical uncertainty	mental uncertainty
Description of uncertainty (how)	occurrence of future event μ is known	truth of a claim μ is unknown
Objective (repeatable)	**Classical** (“objectivist”)	**“Neoclassical”** (pragmatic realist)
Subjective (unrepeatable)		**Bayesian** (subjectivist)

Authors who speak of being 95% confident:

Moore and McCabe:

> We can say we are *95%* ***confident*** that the unknown mean score for all California seniors lies between $\bar{x}$-9 (452) and $\bar{x}$+9 (470). … The statement that ‘we are 95% **confident** that the unknown μ lies between 452 and 480’ is shorthand for saying, ‘We arrived at these numbers by a method that gives correct results 95% of the time.’ ” Moore and McCabe.

Weiss and Hassett,

Ott, Hildebrand and Ott,

Anderson, Sweeney and Williams.

Hamilton

CONCLUSIONS

Distinction between relative frequency basis of probability and the Bayesian subjective notion of probability is more fundamental than the difference between the metaphysical and epistemological differences.

POTENTIALLY AMBIGUOUS STATEMENTS:

[Underscore added to identify a potentially ambiguity]

"The probablity that <u>a confidence interval</u> will include Θ is called the confidence coefficient." Mendenhall, Schaeffer and Wackerly, Mathematical Statistics with Applications, 2nd Ed. 1981. Page 305. Wadsworth, Inc.

OPERATIONAL EQUIVALENCE

GAME: You win the prize if either

1. this particular 90% <u>confidence interval</u> includes the population parameter
2. you randomly draw a red ball from an urn containing 9 red balls and one white ball. (You have a 90% chance/probability of drawing a red ball).

There is no long-term advantage to choosing one over the other.

Thus the classical statement about a confidence interval

is <u>operationally</u> the same as a statement of probability

BEFORE SAMPLE IS TAKEN

THIS KIND OF INTERVAL

Uncertainty is in reality

1. There is a 95% chance that a 95% confidence interval will include μ.

⇓

2. We <u>have 95% confidence</u> (are 95% confident) that <u>this kind of</u> (the next) 95% confidence interval includes μ. ⇒

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1. There is a 100% chance that 95% of these 95% confidence intervals will include μ.

2. We have 100% confidence (are 100% confident) that 95% of these 95% confidence intervals will include μ.

AFTER SAMPLE IS TAKEN

THIS PARTICULAR INTERVAL

Uncertainty is in our minds

1. ~~There is a 95% chance that this 95% confidence interval includes the mean~~

3. We <u>have 95% confidence</u> (are 95% confident) this particular 95% confidence intervals includes μ.

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1. There is a 100% chance that this 95% confidence intervals does or does not include μ.

2. We have 100% confidence (are 100% confident) that this 95% confidence intervals does or does notl include μ.

POINT: LEVEL OF CONFIDENCE PERSISTS BECAUSE THE NEW INFORMATION IS UNINFORMATIVE.

POSTERIOR UNCERTAINTY = PRIOR UNCERTAINTY

NO NEW INFORMATION

Table I: Claims about confidence intervals from a normal population. SE = σ/√n

Confidence Intervals / *Description of uncertainty*	*Context of uncertainty (what)* **Sample has not been drawn.** μ is unknown $\tilde{\bar{x}}$ is random variable	**sample has been drawn statistics are unknown** μ is unknown $\bar{x}_o$ is fixed but unknown	**sample has been drawn Statistics are known** μ is unknown $\bar{x}_o$ is fixed and known
Objective (repeatable)	**Classical** ("objectivist") $P[(\mu_o - 2\,SE) \le \tilde{\bar{x}} \le (\mu_o + 2\,SE)] = .954$	$P[(\mu_o - 2\,SE) \le \bar{x}_o \le (\mu_o + 2SE)] =$ either 0 or 1.	$P[(\mu_o - 2SE) \le \bar{x}_o \le (\mu_o + 2SE)] =$ either 0 or 1
Confidence Strength of belief that $\|\bar{x} - \mu_o\| \le 2$ Std.Errors	⇓ ① **95% Confidence** **[95% Confident]** $\|\tilde{\bar{x}} - \mu_o\| \le 2$ Std.Errors	② ⇒ " **95% Confidence** **[95% Confident]** $\|\bar{x}_o - \mu_o\| \le 2$ Std.Error	③ ⇒ **"Prescriptive"** **95% confidence** **[95% Confident]** $\|\bar{x}_o - \mu_o\| \le 2$ Std.Error
Subjective Bayesian	Probability given by a subjective prior	Probability given by a subjective prior	Posterior given by Bayes rule using sample data and subjective prior

① Confidence is calibrated by means of a probability (Principal principle).

② and ③ This change in facts does not change one's level of confidence.

Table I: Claims about confidence intervals from a normal population: Std Error = σ/√n

Confidence Intervals / *Description of uncertainty*	*Context of uncertainty (what)*		
	Sample not yet drawn. μ_o is known $\tilde{\bar{x}}$ is random variable	**Sample has been drawn** μ_o is unknown $\bar{x}_?$ is fixed but unknown	**Sample has been drawn.** μ_o is unknown $\bar{x}_o$ is fixed and known
Objective Probability	**Classical** $P[(\mu_o - 2\ SE) \le \tilde{\bar{x}} \le (\mu_o + 2\ SE)] = .95$	$P[(\mu_o - 2\ SE) \le \bar{x}_? \le (\mu_o - 2\ SE)]$ equals either 0 or 1.	$P[(\mu_o - 2\ SE) \le \bar{x}_o \le (\mu_o - 2\ SE)]$ equals either 0 or 1
Confidence Strength of belief that $\lvert\bar{x} - \mu_o\rvert \le$ 2 Std.Errors	⇓ ① ***95% confidence [95% confident]*** $\lvert\tilde{\bar{x}} - \mu_o\rvert \le$ 2 Std.Errors	② ⇒ ***95% confidence [95% confident]*** $\lvert\bar{x}_? - \mu_o\rvert \le$ 2 Std.Errors	③ ⇒ **"Neoclassical"** ***95% confidence [95% confident]*** $\lvert\bar{x}_o - \mu_o\rvert \le$ 2 Std.Errors

① Confidence is calibrated by means of a probability (Principal principle).

② No change in our knowledge about the sample statistic

③ Knowing the sample statistic does not change our prediction of the relation between the sample statistic and the population parameter.