The First R - for Reasoning!

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Particularly in the last quarter of a century, statistics has become more pervasive, and increased access to technology also means that more people "do statistics". While society can tolerate more para-statisticians, it cannot afford to be at the mercy of amateurish "dabblers". Citizens must be able to withstand "bad statistics" and appreciate information in increasingly quantitative forms. For example, the media and politicians resort to more data-based arguments, often reaching different conclusions from the same data, as in the BSE/CJD controversy. Members of society are expected to interpret and make life decisions based on school and hospital league tables, and to delight in patient power -- with the right to choose treatments and to have doctors tell them about risks, likely outcomes, etc.

Legal judgements increasingly involve quantitative evidence (e.g. probabilities associated with DNA profiles) that must be weighed up alongside qualitative items such as an accused's claim to have an alibi. Recently, however, the Court of Appeal ruled that ".... the attempt to determine guilt or innocence on the basis of a mathematical formula, applied to each separate piece of evidence, is simply inappropriate to the jury's task". Rather, the Court of Appeal advocated that jurors should use common (un-scientific?) sense. When it comes to statistics and probability, though, "common sense" is often neither common nor sense. Lupton (1892) cited an earlier quote (unfortunately unattributed): "The more ignorant a man is, speaking generally, the more certain he is of the correctness of conclusions derived by invalid methods from incorrect premises."

Very belatedly, some recognition of the need for statistical literacy in order to combat such ignorance has emerged. In common with trends in most industrialised countries, systematic statistical instruction has been introduced into the National Curriculum for Mathematics for all pupils aged 5 - 16 in England and Wales (and Northern Ireland). However, "Statistics for All" policies do not necessarily yield "Statistical Literacy for All". Research would suggest that the simple answer to the question "Can a mathematically educated person be statistically illiterate?" is "Yes". Statistical and probabilistic misconceptions persist in people who have followed mathematics courses even to quite high levels. Indeed, Fischbein and Schnark (1997) have reported finding that some misconceptions seem actually to worsen with exposure to more mathematical training.

Recently, I have been working with a group of trainee lawyers, all graduates, a third of whom had studied mathematics to beyond compulsory levels (i.e. to 'A'-level and higher). While there was a little evidence to suggest that some aspects of likelihood and probability judgements did worsen with more mathematics training, more particularly they did not improve. The lawyers were all as bad as each other, irrespective of their mathematical backgrounds. A comparison group of post-graduate students who were all practising statistics teachers were regrettably also not infallible on the same test items!

The converse of the question "Can a mathematically-educated person be statistically illiterate?" is also pertinent, because of the way in which most people's statistical education is subsumed under their school mathematics curriculum. Would we be prepared to say that someone was mathematically educated if he/she was statistically illiterate? It should be a contradiction in terms, but "Statistics for All" policies tend to emphasise knowledge of techniques, while what we really need to develop are the skills, understanding and inclination to use such techniques, i.e. "Statistical Literacy for All". My two criteria for assessing whether statistical education has succeeded would be that its recipient was able to function effectively in a world of uncertainty, and had the skills to summarise and represent information (be it qualitative or quantitative) for him/herself and others.

Much of the research into probabilistic misconceptions addresses error types, e.g. over-reliance on the availability heuristic (based on the ease with which things can be brought to mind), succumbing to the representativeness fallacy (applying population reasoning to sample situations), etc. These studies may suggest remediation strategies, but what the teacher needs is prevention rather than cure. Glickman (Hawkins et al, 1982) argued that more useful approaches might involve attacking the students' failure to formulate and/or failure to enumerate uncertain outcomes, i.e. placing more emphasis on model construction rather than manipulation. More graphical approaches to teaching probability may well help to achieve this. See, for example, Tomlinson and Quinn (1997) on the use of Intersection Tables and Tree Diagrams for conveying ideas of conditional probability. Also, though, see Abele (1983) and Watson and Moritz (1997) on students' own inventions of graph forms as useful precursors to conventional formats, and Pereira-Mendoza and Mellor (1991) on children's misconceptions about conventional graph forms, for example their belief that graphs must mean something "causal".

Reading, writing and arithmetic are of little or no use if they are not underpinned with reasoning. This first R -- for reasoning -- is what makes statistics more than just meaningless 'rithmetic, but the "back to basics" approach tends to highlight, and reward, just those activities that are outside the realms of reason! Once we add in reasoning, though, we have the beginnings of statistical literacy, an obvious application of the three, or rather four, R's. Conversely, in teaching and developing statistical literacy, we contribute much to the proper development of the individual activities of reading, 'riting and 'rithmetic -- the literacy and numeracy that Society demands. Statistical literacy certainly requires a synergy of all the 4 R's, but these must be manifest in a rather broader range of activities, including, at least: literacy; numeracy; visualisation; graphicacy; pattern perception; (re)presentational skills for qualitative or quantitative data; fluency in language and principles of statistics and probability; appreciation of chance and randomness; ability to operate in the real (multivariate) world; ability to construct as well as to manipulate (probabilistic) models; ability to communicate, comprehend and critically evaluate arguments couched in statistical or probabilistic terms; appreciation of investigative rigour; computer literacy.

Most of the items in this seemingly daunting list are more important building blocks than the tools that we currently tend to give (and which most of our students soon forget). These skills and understanding are eminently more usable, generalisable and transferable. Typically, though, mathematics education does not result in the necessary interface of cognitive activities. For example, when some mathematics educators talk of graphicacy they focus on how, rather than why, to draw a graph. Maybe statistical education is not best served within the mathematics curriculum? Maybe statistics is too multi-faceted, and relies on reasoning processes that are too much at odds with traditional school views of "doing mathematics"? If this is the case, a different term may be needed in order to ensure that statistical literacy features in the arguments about how to produce a numerate and literate society. How about "informacy", "statisticacy", or "statomeracy", for example?

Let us assume that what we want to achieve is "statistical literacy" for all, and "statistical literacy plus" for some. If we take a dimension of specialists from users to producers of statistics, we can see that non-specialists and specialists in other subjects would not necessarily need highly developed "production" (i.e. "literacy plus") skills. We do, however, need non-statisticians to understand the principles (and language) of statistics, although a statistician who cannot talk to the non-specialist in a language that the latter will understand is just as statistically illiterate as someone who is unversed in statistics. Much of the statistical activity in the workplace is collaborative, even though our education assessment system tends towards training individual competitiveness. We cannot overlook the importance of communication skills as a vital component of being statistically educated. Otherwise, how will specialists in other areas work effectively with specialists in statistics, or appreciate what statisticians and their discipline can contribute?

Typically, education and assessment practices tend to separate the "plus" from the literacy, and to emphasise the former, delivering techniques, techniques, and more techniques. However, a common complaint of employers of recent graduates (as highlighted by the MEANS project -- Matching Education, Assessment and Employment Needs in Statistics) is that recruits from more statistical/quantitative specialities are too technique-oriented, with not enough common sense, or feel for applying their knowledge to real situations, and not enough team-player/consultancy skills. Non-specialists in statistics, on the other hand, often seem to have developed little more than statisticaphobia!

If there is no guarantee that more and more "plus" necessarily turns into statistical literacy, it is time for us to stop hitting our heads against a brick wall, and to engage in more radical rethinking about our approach to statistical education. Statistical education has evolved to where it is at present, but there is a case for saying that this is not the right starting point for where it should be going in the future. David Moore (1997) refers to the Professional's Fallacy -- this is what I was taught, and the way that I was taught it, so it must be the right (and by implication the only) way to proceed. But many of our students are not "us" a few years down the line. Statistics has changed, and so have the pool of students, and their needs.

If they are to break with the existing content and approaches to teaching statistics, however, most of our teachers need training in statistical pedagogy and many need training in statistics. Those who lack statistical process understanding are ill-equipped to adopt more constructivist teaching approaches, abandoning technique-based progression through a statistics syllabus in favour of more discursive teaching strategies. The move from "Sage on the Stage" to "Guide on the Side" is not an easy one to make. Furthermore, guides must still "stage-manage". Skills do not rub off on students just because they encounter real data, and teachers need help in knowing how to develop the relevant data-handling skills in their students. Most of the support for teachers of statistics in England, though, has been (and still is) at the in-service rather than pre-service level. It has also taken some rather ad hoc forms, e.g. pupils' competitions with feedback for teachers, and has therefore tended to reach only those who are already converted.

Content and assessment often seem to prevail over other considerations of how to promote effective learning. Certainly assessment seems to be increasingly the major factor for students, while content often seems to be the sole concern of teachers, given that what will be taught is governed by the (ease of) assessment tail wagging the teaching dog. It is not just teachers who need help to change their approaches. Curriculum and syllabus planners, and members of examination boards must also be educated in the real nature of statistical understanding, and how it is best taught and assessed. Teachers cannot be expected to make changes in their teaching methods unless assessment methods also change.

Course content is sometimes a form of academic protection -- i.e. if the course content is not difficult (obscure or inaccessible enough) then its teachers may not feel academically credible to their colleagues. However, more complex/sophisticated computational techniques are not necessarily more intellectually or academically demanding. It is like ballroom dancing -- grasping the basic principles of movement and balance is what is important (and difficult) -- after that, it's all just steps! The prime function of teaching is communication. It is therefore our job to find ways of teaching the principles or basics of our subject, not of limiting those who can avail themselves of this understanding.

In its earliest days, developments in statistical education were hampered by the lack of a body of research (systematic or otherwise) on which to base advice to teachers. Four main areas of research can now be identified, however, concerned with: statistical understanding; probabilistic understanding; visualisation; and teaching methods and materials. We do not have enough knowledge about how to teach statistics more effectively, but the real problem now is that the knowledge that we do have is not always implemented. Dissemination must reach those who have not yet espoused an interest in statistical education, as well as those who have.

We should take none of our current teaching practices for granted. The effectiveness, or otherwise, of most of them has never been empirically evaluated. However, there is not only a need for original empirical work. There is considerable need for the existing research literature to be rationalised. For example, the gulfs between cognitive and classroom-based (and developmental) research must be bridged so that findings in one area can inform work in the others. There is plenty of scope for teachers to undertake relevant research to this end in their own classrooms. My personal perception of research priorities, is as follows:

- Conceptions/misconceptions/reasoning -- statistical as well as probabilistic
- Visualisation and graphicacy, not forgetting data tables
- Technology -- design and uses
- Content/approaches -- emphasis on model construction and use
- How to achieve skills training
- Assessment methods -- including their implementation

The really big research question that faces us, though, is how to produce statistically literate citizens. In 1892, Sydney Lupton stated:

"If we accept the definition of Laplace, that the theory of probability consists in 'common sense reduced to calculation', everyone will admit that instruction in both faculties is well worthy of the attention of the educationalist." What would be the effect on our teaching strategies and outcomes if we were to focus (as Lupton advocates) on common sense as much as on calculation? The expression "back to basics" is misleading. Reasoning must be at least as basic as reading, 'riting and 'rithmetic, but if we are to achieve our aim of "Statistical Literacy for All" we will have to rethink our approach to teaching statistics, going forward to this particular basic. How can we make sure that common sense or reasoning feature specifically in our objectives? This is the really big challenge that faces statistical educators and researchers today.

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