MEDIAN OVERLAP, MISCLASSIFICATION AND R^2

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Abstract: The median overlap is an extremely useful descriptive statistic. It compares the relationship between two univariate distributions involving a continuous variable (e.g., heights of males and females). The median overlap is the percentage of subjects in one group having a value that goes past the median value for subjects in the other group (e.g., percentage of females whose heights exceed the median height for males). When the variable of interest is normally distributed with equal standard deviations in both groups, *the median overlap is closely related to the chance that the comparative relationship between a random subject from each group will be misclassified*. (If the median overlap is 16% for heights of males and females, the chance of a male being shorter than a female is 24%.) Students should be exposed to the chance of misclassification so as to improve their understanding of the discriminatory power of a particular binary variable. The median overlap determines the value of R^2 in an OLS regression. The rate of misclassification is shown to be modeled as 43% - .79*R^2. The certainty in making a correct comparison is modeled as 57% + .78 R^2. An R^2 of about 50% gives one about 90% certainty in making a comparative prediction.

I. INTRODUCTION:

Often our goal is to make predictions. To improve the quality of our predictions, we use the values of related variables. This is commonly done involving discrete classification variables (tables) and continuous variables (regression).

Consider a group of subjects having a quantitative property (e.g., height). Suppose these subjects can be divided into two groups by a binary variable (e.g., sex) having two values (e.g., male and female). Suppose the two distributions have medians that differ by a certain amount (X). Given this, we can identify the proportion of the one population that has values of the continuous variable that overlap past the median of the other distribution (e.g., the percentage of females who are taller than the median male). This percentage is called the median overlap (Bell Curve, p. 49).

For illustration, suppose that both distributions were normal with equal standard deviations (s) and a separation distance, X. In this case the median overlap would be CDF(-X/s). So if X is 3" and the standard deviation for height in both sexes is also 3", then the median overlap should be about 16%. This is shown in the following picture.

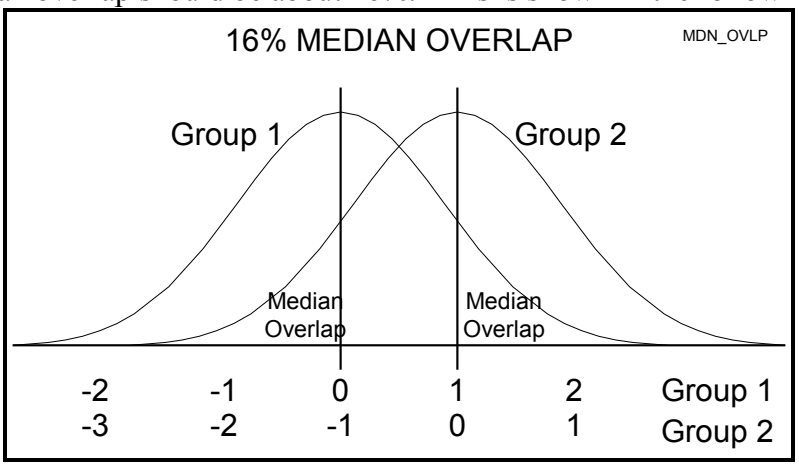

Figure 1: Median Overlap

There are two ways of expressing the median overlap in terms of probabilities.

1. The median overlap is the chance that a random member of one group will be above (below) the median of the other group.
2. If the variable of interest is normally distributed with equal standard deviations, *then the median overlap is always less than the chance of misclassification using group membership to predict the relationship between random members from each group*.

The truth of the first expression is a simple consequence of the nature of the median overlap and the equality of opportunity under a random choice. The truth of the second expression requires proof.

To understand what is being asserted, consider the following cases:

Sex and height. Assume that 16% of all females are taller than the median height of the males (a 16% median overlap). In this case, there is a 16% chance that a random male will be shorter than the average female. If heights are normally distributed, there is *at least* a 16% chance that a random male will be shorter than a random female.

Handedness and IQ. Assume that left-handers are slightly smarter than right-handers. Specifically, assume that 40% of all left handers have an IQ that is below the median IQ of the right handers (a 40% median overlap). In this case there is a 60% chance that the IQ of a random left hander will be higher than the IQ of a random right hander. If these IQs are normally distributed with equal standard deviations, then there is *at least* a 60% chance that the IQ of a random left-hander will be greater than the IQ of a random right-hander.

II. PROOF: If two groups (males and females) have a common property (e.g., height) which is normally distributed, and if subjects are picked randomly from both groups, then the resulting distribution will be a bivariate normal. The correlation coefficient between the random selections from both groups will be zero. If the means for both groups are identical, then the joint center would be located on the 45^o line. On one side of this line are those subjects which are properly classified (male is taller than female); on the other side of this line are those subjects that are misclassified (male is shorter than female). In such a case, the proportion of the bivariate normal involving the misclassification is 50% -- half of the bivariate distribution.

Now consider Figure 2 in which the medians for the two groups are unequal. The joint center will be to one side (or the other) of this 45^o line. Those subjects who are misclassified based on their sex are located on the side of the 45^o line opposite the side containing the joint center.

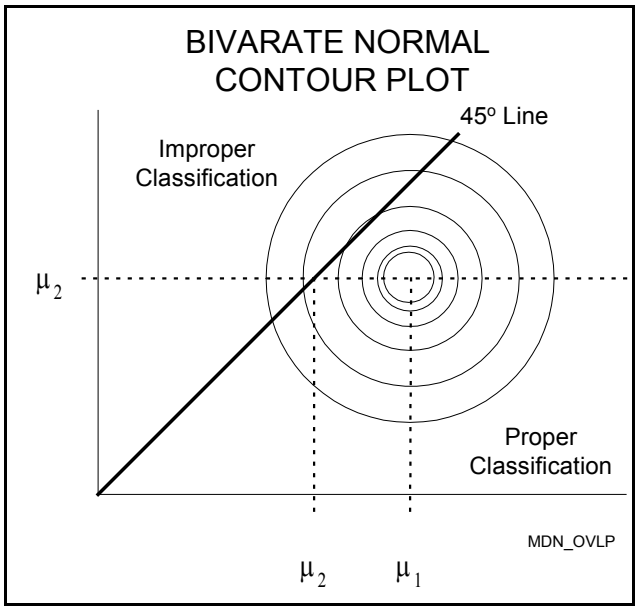

Figure 2: Classification Region on Bivariate Normal

Since the heights of males and females selected randomly should be uncorrelated ($\rho = 0$), it follows that a slice in any direction will yield a normal distribution. Each slice has a normal distribution with the same standard deviation. Consider all the slices perpendicular to the 45º line. *Each perpendicular slice is cut off by the 45º line at the same value of Z (say Z_p).* Thus, the proportion of the volume on the wrong side of the 45º line (equal heights) is simply the p-value associated with that value of Z_p. The value of Z_p is $Z/\sqrt{2}$ where $Z = (\mu_2 - \mu_1) / s$. The median overlap is determined by the tail above Z; the rate of misclassification is determined by the tail above $Z/\sqrt{2}$. Thus, the percentage of misclassifications is always greater than the median overlap.

Figure 3 illustrates the relation between the misclassification rate and the median overlap.

The maximum difference is 8.3 percentage points; this occurs when the median overlap is 12%. Thus, median overlap ≤ misclassification rate ≤ (median overlap + 8 percentage points).

If the two groups have equal numbers then the overall group median is at Z/2. If one calculates [CDF(Z/2) + CDF(Z)]/2 = [(Joint Median Overlap) + (Opposite group median overlap)]/2, this is within 1 percentage points of the misclassification rate for all values of Z for $Z \leq 2$. The misclassification rate is approximately equal to the average of two median overlap rates.

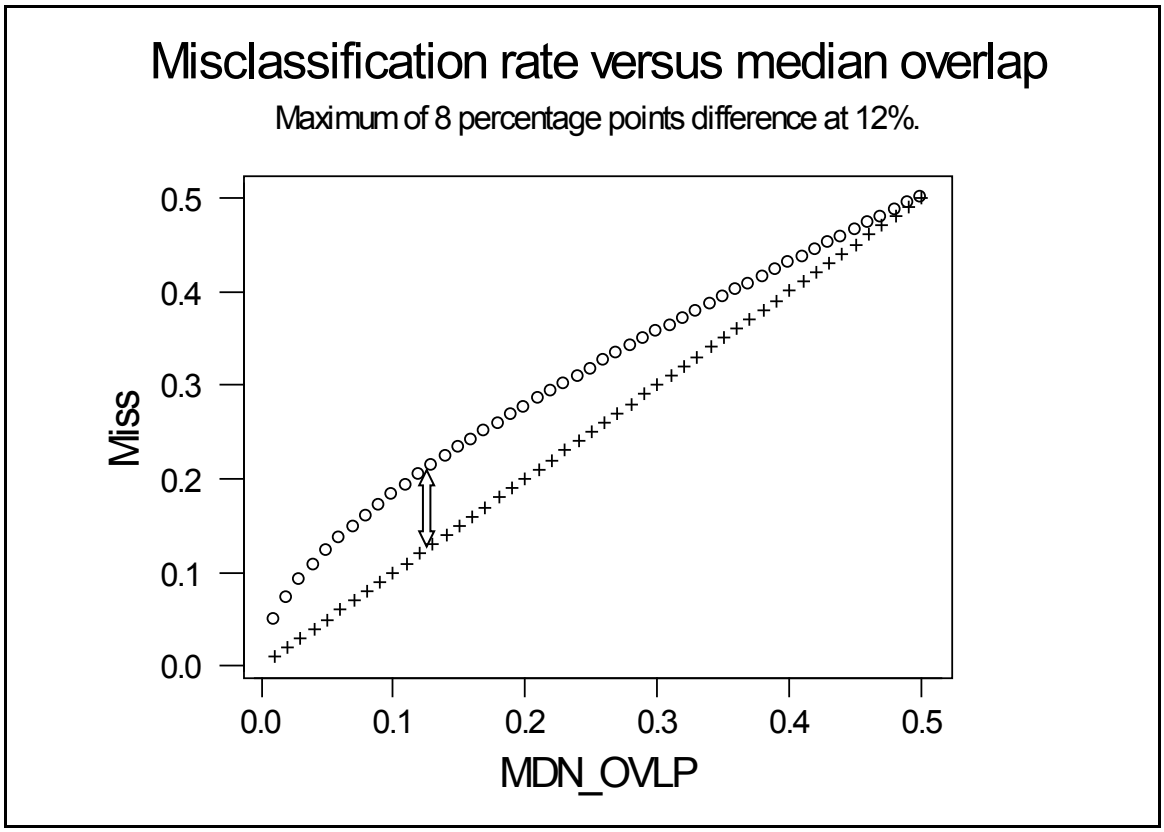

Figure 3: Misclassification Rate vs. Median Overlap

As the median overlap decreases, the rate of misclassification decreases. These two variables are positively associated and closely associated.

III. EXAMPLES:

According to the Bell Curve (p. 49), there is a 7% median overlap in IQ between graduates of high school (who don't complete college) and graduates of college (Z=1.48). Suppose one selects a random college graduate and a random high-school graduate who didn't complete college. Assuming these IQs are normally distributed with equal standard deviations, there is a 15% chance of misclassification since $Z_p = 1.05$. Thus there is an 85% chance that the IQ of a random college graduate exceeds that of a random high school graduate.

According to the Bell Curve (p. 49), there is a 21% median overlap in IQ between those college graduates who have obtained a Ph.D., M.D., or LL.B., and those college graduates who have not (Z= .81). Assuming these IQs are normally distributed with equal standard deviations, there is a 28% chance of misclassification since $Z_p = 0.57$. Thus there is a 72% chance that the IQ of a random student receiving the advanced degree is higher than the IQ of a random student who completed college but did not receive the advanced degree.

IV. ERROR REDUCTION:

These probabilities of correct forecasts seem quite high. Remember that even if there is no predictive power for a given binary variable, one will still be right 50% of the time due strictly to chance. An alternate way to measure improvement is to calculate how much of the original uncertainty (50%) was removed for various Misclassification Rates (MR). The reduction in error is 100% (50% - MR) / MR.

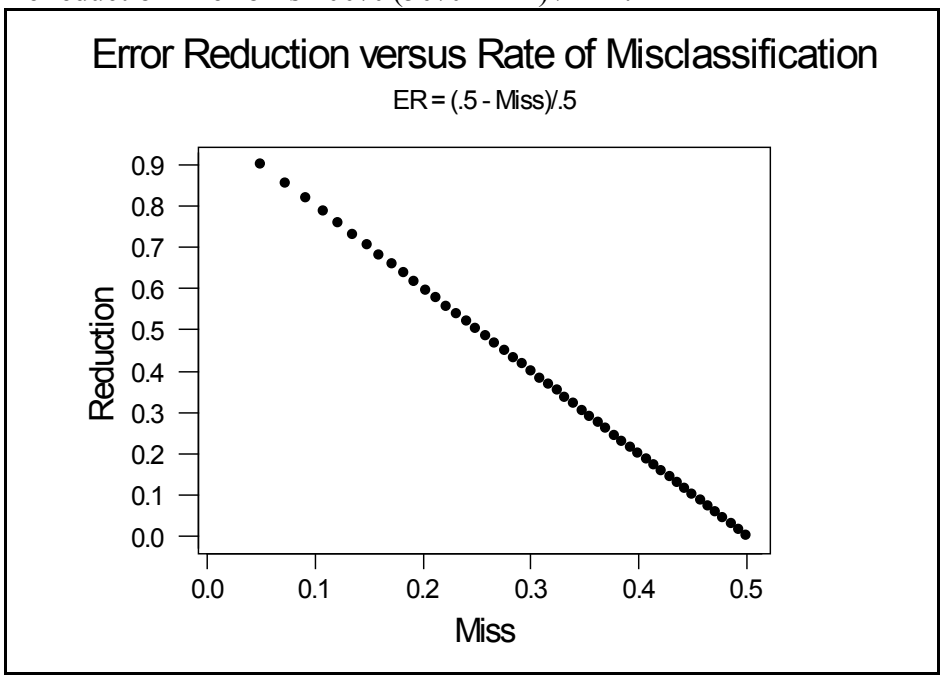

Figure 4: Error Reduction vs Misclassification Rate

The median overlap is closely related to the rate of misclassification. Thus, as the median overlap decreases, the rate of reduction in misclassification error increases.

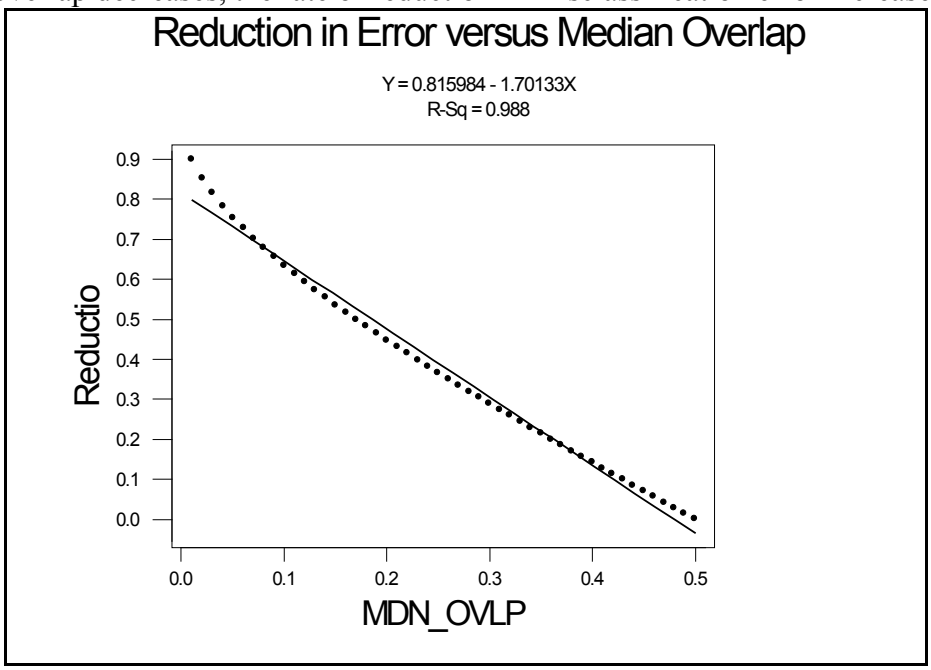

Figure 5: Error Reduction vs Median Overlap

V. R^2:

In an ordinary least-squares regression, R^2 is the percentage of variance eliminated by using the model. Let Z measure the separation distance between the medians of the two groups measured in standard deviations. When the variances of the two groups are equal and when the numbers in the two groups are equal, it is shown in the appendix that:

$Z^2 = (\mu_2 - \mu_1)^2 / s^2$.

$R^2 = (Z^2/4)/ [(Z^2/4 + 1)] = 1/ [(1 + 1/(Z^2/4)]$

An OLS regression using Median Overlap to predict R^2, gives approximately

$R^2 = 41\% - \text{Median Overlap}$

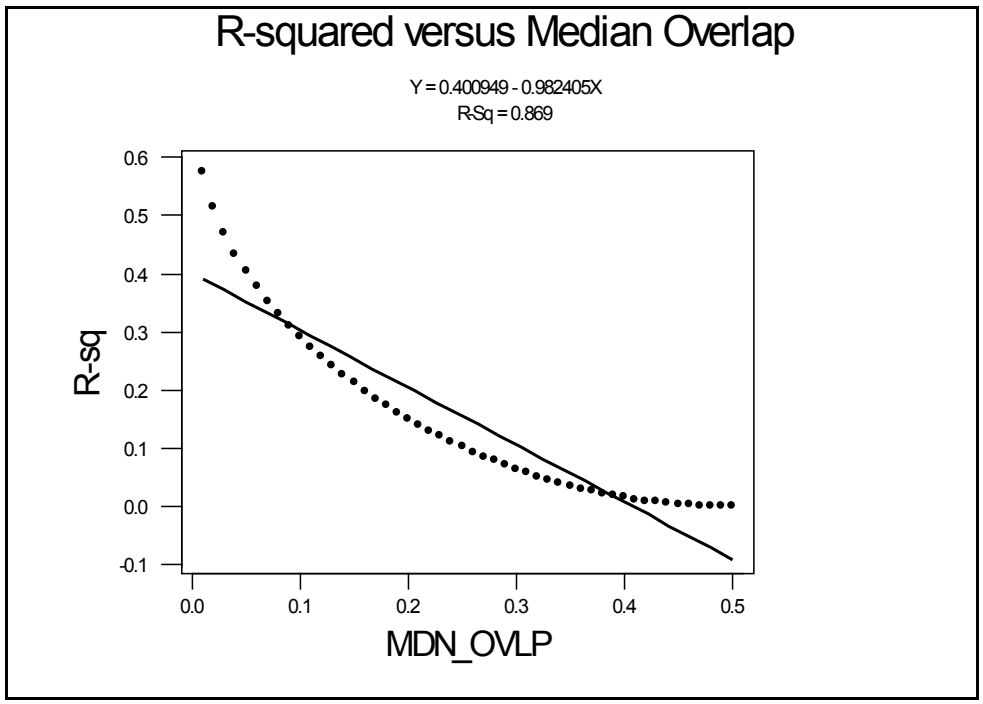

Figure 6: R-squared versus Median Overlap

An OLS regression using R^2 to predict Median Overlap, gives approximately

Median Overlap = 39% - .89 R^2

The proportion misclassified can be estimated using OLS regression on R^2:

Proportion Misclassified = 0.427 - 0.785 R^2.

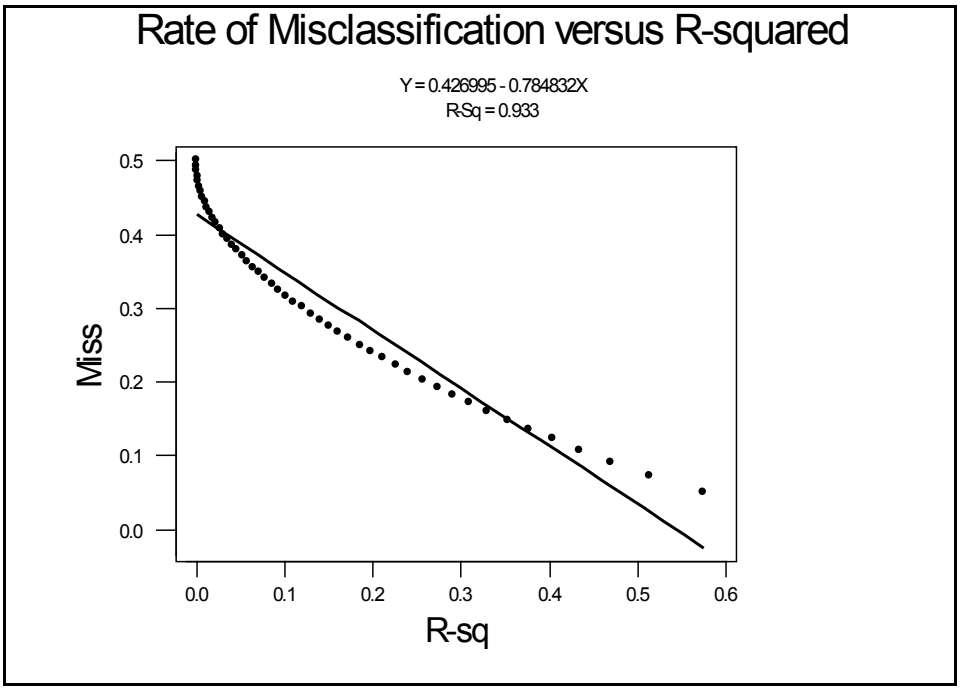

Figure 7: Misclassification Rate vs R-Squared

We can also model the probability of being correct for our comparison.

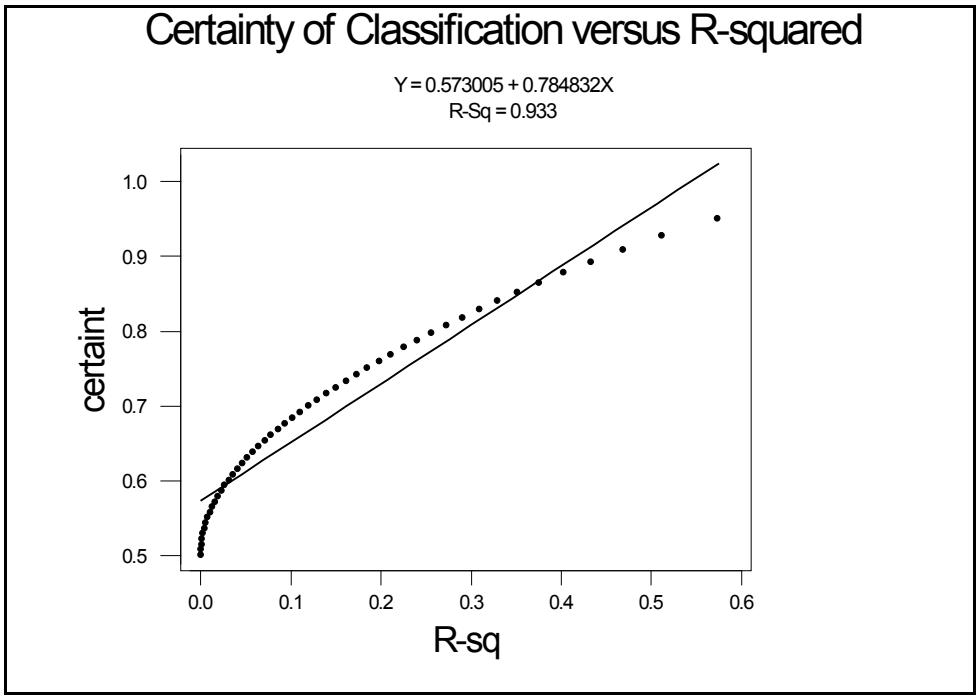

Figure 8: Classification Certainty vs R-Squared

An R^2 of 0.5 takes one to a probability of about 90% in making a relative classification correctly. Thus, it doesn't take a very high R^2 to make accurate predictions involving a simple comparison.

VI. ERROR REDUCTION AND R^2

We can compare R^2 with the rate of error reduction. This is shown in the following figure.

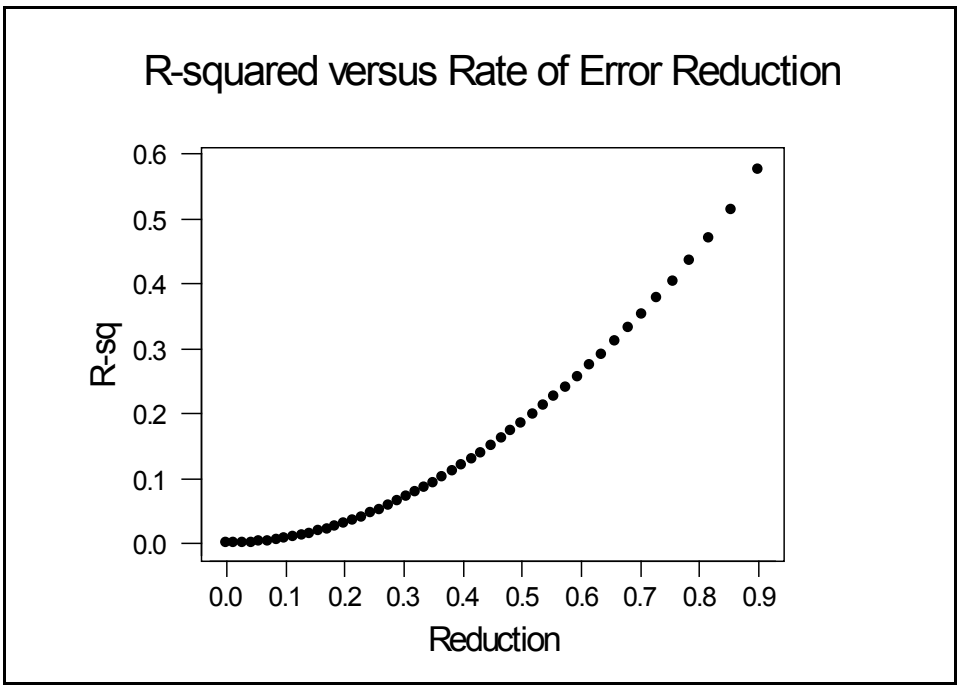

Figure 9: R-Squared vs Error Reduction Rate

VII. SUMMARY

We are justified in using the value of a binary variable (male vs. female) to make an inferential comparison of a related quantitative variable (height) when the median overlap is small. To obtain 80%, 90% or 95% rates of correct prediction, the median overlap must be of 12%, 5% and 1% respectively.

It doesn't take a very high value of R^2 to have a 90% certainty in making a simple ordinal comparison (height of male > height of female). One reason is that by chance alone, one has a 50% chance of making such a comparison correctly. So an R^2 of zero gives a 50% chance of a correct comparison.

To obtain a 90% chance of making a correct comparison, one needs a median overlap of no more than 12% or an R^2 of at least 37%.

APPENDIX: RELATION BETWEEN STANDARD DEVIATIONS
Thanks to Gerald Kaminski:

Consider two distributions, $X(\mu_1, \sigma_1)$ and $X(\mu_2, \sigma_2)$ having n_1 and n_2 values respectively. If these two distributions are mixed together, then the combined distribution has a center μ_0 and a standard deviation σ_0. Assume $\mu_1 < \mu_2$.

$\mu_0, = (n_1 \mu_1 + n_2 \mu_2)/(n_1+n_2)$ Eq 1.

$\sigma_0^2 = \sum (x - \mu_0)^2 /(n_1+n_2)$ Eq 2

Now divide this into two parts – one for each of the two distributions:

$\sigma_0^2 = \sum [(x - \mu_1) + (\mu_1 - \mu_0)]^2 /(n_1+n_2) + \sum [(x - \mu_2) + (\mu_2 - \mu_0)]^2 /(n_1+n_2)$ Eq 3.

where each part includes only the data points from that group. Expanding, we get

$\sigma_0^2 = \sum [(x - \mu_1)^2 + 2(x - \mu_1)(\mu_1 - \mu_0) + (\mu_1 - \mu_0)^2] /(n_1+n_2) +$ Eq 4

$\sum [(x - \mu_2)^2 + 2(x - \mu_2)(\mu_2 - \mu_0) + (\mu_2 - \mu_0)^2] /(n_1+n_2)$

Note that the cross-products sum to zero.

$s_1^2 = (x - \mu_1)^2/n_1$ and $s_2^2 = (x - \mu_2)^2/n_2$ for the respective groups. Eq 5.

$\sigma_0^2 = \{n_1 [s_1^2 + (\mu_1 - \mu_0)^2] /(n_1+n_2)\} + \{n_2 [s_2^2 + (\mu_2 - \mu_0)^2] /(n_1+n_2)\}$ Eq 6.

If $n_1 = n_2$, then

$\mu_0 = (\mu_1 + \mu_2)/2$ so that $\mu_0 - \mu_1 = \mu_2 - \mu_0$. Thus, $\mu_2 - \mu_1 = 2(\mu_2 - \mu_0)$ Eq 7.

$\sigma_0^2 = \{[s_1^2 + (\mu_1 - \mu_0)^2] / 2\} + \{[s_2^2 + (\mu_2 - \mu_0)^2] / 2\}$ Eq 8.

$\sigma_0^2 = [s_1^2 + s_2^2]/2 + [(\mu_1 - \mu_0)^2 + (\mu_2 - \mu_0)^2] / 2$ Eq 9.

$(\mu_1 - \mu_0)^2 + (\mu_2 - \mu_0)^2 = [2\mu_1 - (\mu_1 + \mu_2)]^2 + (2\mu_2 - (\mu_1 + \mu_2)]^2 = 2(\mu_2 - \mu_1)^2$ Eq 10.

$\sigma_0^2 = [s_1^2 + s_2^2]/2 + (\mu_2 - \mu_1)^2$ Eq 11.

If $s_1 = s_2 = s$, and $n_1 = n_2$, then

$\boxed{\sigma_0^2 = s^2 + [(\mu_2 - \mu_1)/2]^2}$ Eq 12.

Let $Z = (\mu_2 - \mu_1) / s$. Thus, $s^2 = (\mu_2 - \mu_1)^2 / Z^2$ Eq 13.

$\sigma_0^2 = (\mu_2 - \mu_1)^2[1/4 + 1/Z^2] = s^2Z^2 [1/4 + 1/Z^2] = s^2(Z^2/4 + 1)$ Eq 14.

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$R^2 = (\sigma_0^2 - s^2)/ \sigma_0^2 = [s^2(Z^2/4 + 1) - s^2]/ [s^2(Z^2/4 + 1)]$ Eq 15.

$\boxed{R^2 = (Z^2/4)/ [(Z^2/4 + 1)] = 1/ [(1 + 1/(Z^2/4)]}$ Eq 16.

For small Z, $R^2 = Z^2/4$. For large Z, $R^2 = 1 - 1/(Z^2/8)$. Eq 17.

For s = 1 and $\mu_2 - \mu_1 = 1$, Z = 1 and $R^2 = 20\%$. Here the median overlap is 16%.

For s = 1 and $\mu_2 - \mu_1 = 2$, Z = 2 and $R^2 = 50\%$. Here the median overlap is 2.5%.

For s = 1 and $\mu_2 - \mu_1 = 3$, Z = 3 and $R^2 = 69\%$. Here the median overlap is 0.5%.

APPENDIX:

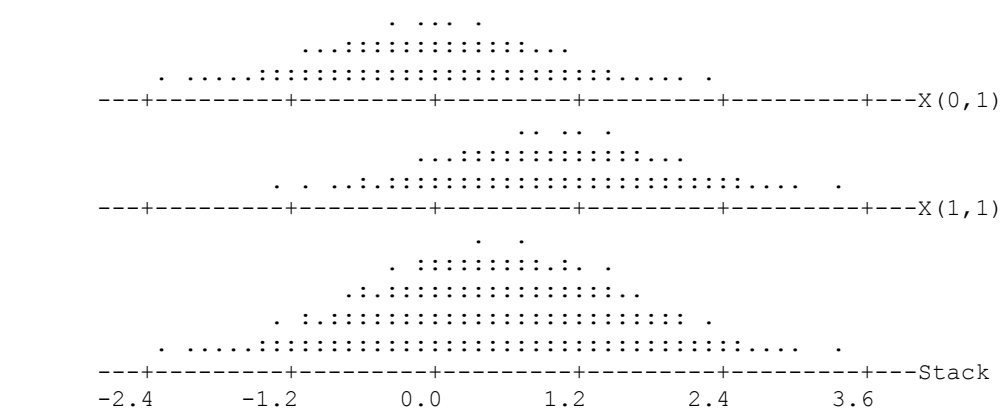
```
MTB > Stack c2 c3 c4;
SUBC > subscripts c5.

MTB > desc c2 c3 c4
Descriptive Statistics

Variable        N     Mean   Median  Tr Mean    StDev  SE Mean
X(0,1)         99   0.0000   0.0000   0.0000   0.9652   0.0970
X(1,1)         99   1.0000   1.0000   1.0000   0.9652   0.0970
Stack         198   0.5000   0.5000   0.5000   1.0854   0.0771

Variable      Min      Max       Q1       Q3
X(0,1)    -2.3263   2.3263  -0.6745   0.6745
X(1,1)    -1.3263   3.3263   0.3255   1.6745
Stack     -2.3263   3.3263  -0.2598   1.2598

MTB > dotplot c2 c3 c4;
SUBC> same.
Character Dotplot
```




```
MTB > brief 1
MTB > regress c4 1 predictor c5
Regression Analysis

The regression equation is
Stack = - 1.00 + 1.00 Subscript

Predictor       Coef       StDev          T        P
Constant     -1.0000      0.2169      -4.61    0.000
Subscrip      1.0000      0.1372       7.29    0.000

S = 0.9652      R-Sq = 21.3%     R-Sq(adj) = 20.9%

Analysis of Variance

Source       DF          SS          MS         F        P
Regression    1      49.500      49.500     53.14    0.000
Error       196     182.588       0.932
Total       197     232.088
```

```
MTB > Stack c7 c3 c8;
SUBC > subscripts c9.


MTB > desc c7 c3 c8
Descriptive Statistics

Variable        N     Mean   Median  Tr Mean    StDev  SE Mean
X(-1,1)        99  -1.0000  -1.0000  -1.0000   0.9652   0.0970
X(1,1)         99   1.0000   1.0000   1.0000   0.9652   0.0970
Stack2        198   0.0000   0.0000   0.0000   1.3899   0.0988

Variable      Min      Max       Q1       Q3
X(-1,1)   -3.3263   1.3263  -1.6745  -0.3255
X(1,1)    -1.3263   3.3263   0.3255   1.6745
Stack2    -3.3263   3.3263  -1.0511   1.0511


MTB > dotplot c7 c3 c8;
SUBC> same.
Character Dotplot

                        . .. ..
                   ...:::::::::::::::...
          .  ....:::::::::::::::::::::::::::::.:.. . .
           -------+---------+---------+---------+---------+---------X(-1,1)
                                       .. .. .
                                  ...:::::::::::::::...
                        . . ..:.:::::::::::::::::::::::::::....  .
           -------+---------+---------+---------+---------+---------X(1,1)
                        ...: :::.::::::::.::: :...
                   ...:::::::::::::::::::::::::::::::...
          .  ....:::::::::::::::::::::::::::::::::::::::::::....  .
           -------+---------+---------+---------+---------+---------Stack2
               -2.4      -1.2       0.0       1.2       2.4       3.6


MTB > Regress 'Stack2' 1 'Subscr2';
SUBC>   Constant.

Regression Analysis


The regression equation is
Stack2 = - 3.00 + 2.00 Subscr2

Predictor       Coef       StDev          T        P
Constant     -3.0000      0.2169     -13.83    0.000
Subscr2       2.0000      0.1372      14.58    0.000

S = 0.9652      R-Sq = 52.0%     R-Sq(adj) = 51.8%

Analysis of Variance

Source       DF          SS          MS         F        P
Regression    1      198.00      198.00    212.54    0.000
Error       196      182.59        0.93
Total       197      380.59
```

COMPARE MEDIAN OVERLAP AND MISCLASSIFICATION %.

```
MTB > Set c1
DATA>   1( .01 : .5 / .01 )1
DATA>   End.
MTB > InvCDF c1 c2;
SUBC>   Normal 0.0 1.0.
MTB > let c3 = .707*c2
MTB > CDF c3 c4;
SUBC>   Normal 0.0 1.0.


MTB > regress c4 1 c1;
SUBC> predict .5.
Regression Analysis

The regression equation is
%Miss = 0.0920 + 0.851 MDN_OVLP

Predictor       Coef       StDev          T        P
Constant    0.092008    0.004009      22.95    0.000
MDN_OVLP     0.85067     0.01368      62.17    0.000

S = 0.01396     R-Sq = 98.8%     R-Sq(adj) = 98.7%

Analysis of Variance

Source       DF          SS          MS         F        P
Regression    1     0.75348     0.75348   3865.15    0.000
Error        48     0.00936     0.00019
Total        49     0.76284

Unusual Observations
Obs  MDN_OVLP      %Miss        Fit  StDev Fit   Residual    St Resid
  1     0.010    0.05001    0.10051    0.00389   -0.05050      -3.77R
  2     0.020    0.07325    0.10902    0.00377   -0.03577      -2.66R

R denotes an observation with a large standardized residual

     Fit  StDev Fit       95.0% CI             95.0% PI
 0.51734    0.00389   ( 0.50952, 0.52516)  ( 0.48819, 0.54649)
```

```
MTB > desc c1 c12

Variable        N     Mean   Median  Tr Mean    StDev  SE Mean
MDN_OVLP       50   0.2550   0.2550   0.2550   0.1458   0.0206
R-sq           50   0.1504   0.0979   0.1355   0.1536   0.0217

Variable      Min      Max       Q1       Q3
MDN_OVLP   0.0100   0.5000   0.1275   0.3825
R-sq       0.0000   0.5750   0.0219   0.2448

MTB > regress c12 1 c4
The regression equation is
R-sq = 0.518 - 1.19 Miss

Predictor       Coef       StDev          T        P
Constant     0.51788     0.01525      33.97    0.000
Miss        -1.18942     0.04582     -25.96    0.000

S = 0.04002     R-Sq = 93.3%     R-Sq(adj) = 93.2%

Analysis of Variance
Source       DF          SS          MS         F        P
Regression    1      1.0792      1.0792    673.78    0.000
Error        48      0.0769      0.0016
Total        49      1.1561

Unusual Observations
Obs      Miss       R-sq        Fit  StDev Fit   Residual    St Resid
  1     0.050    0.57501    0.45840    0.01314    0.11661       3.08R
  2     0.073    0.51326    0.43076    0.01219    0.08250       2.16R
R denotes an observation with a large standardized residual

MTB > regress c4 1 c12
The regression equation is
Miss = 0.427 - 0.785 R-sq

Predictor       Coef       StDev          T        P
Constant    0.426995    0.006467      66.02    0.000
R-sq        -0.78483     0.03024     -25.96    0.000

S = 0.03251     R-Sq = 93.3%     R-Sq(adj) = 93.2%

Analysis of Variance
Source       DF          SS          MS         F        P
Regression    1     0.71211     0.71211    673.78    0.000
Error        48     0.05073     0.00106
Total        49     0.76284

Unusual Observations
Obs      R-sq       Miss        Fit  StDev Fit   Residual    St Resid
  1     0.575    0.05001   -0.02429    0.01364    0.07430       2.52RX
  2     0.513    0.07325    0.02417    0.01189    0.04908       1.62 X
 49     0.000    0.49293    0.42687    0.00646    0.06606       2.07R
 50     0.000    0.50000    0.42699    0.00647    0.07301       2.29R
R denotes an observation with a large standardized residual
X denotes an observation whose X value gives it large influence.
```

```
MTB > let c5 = c4 - c1
MTB > let c6 = c4/c1
MTB > let c7 = (c4-c1)/c1
MTB > print c1 c2 c4 c5 c6 c7
```

Row	MDN_OVLP	Z	Miss	Diff	Ratio	RelDiff
1	0.01	-2.32635	0.050013	0.0400129	5.00129	4.00129
2	0.02	-2.05375	0.073251	0.0532508	3.66254	2.66254
3	0.03	-1.88079	0.091805	0.0618052	3.06017	2.06017
4	0.04	-1.75069	0.107907	0.0679072	2.69768	1.69768
5	0.05	-1.64485	0.122433	0.0724328	2.44866	1.44866
6	0.06	-1.55477	0.135835	0.0758350	2.26392	1.26392
7	0.07	-1.47579	0.148385	0.0783852	2.11979	1.11979
8	0.08	-1.40507	0.160261	0.0802611	2.00326	1.00326
9	0.09	-1.34076	0.171587	0.0815867	1.90652	0.90652
10	0.10	-1.28155	0.182453	0.0824528	1.82453	0.82453
11	0.11	-1.22653	0.192928	0.0829284	1.75389	0.75389
12	0.12	-1.17499	0.203067	0.0830671	1.69223	0.69223
13	0.13	-1.12639	0.212912	0.0829118	1.63778	0.63778
14	0.14	-1.08032	0.222497	0.0824974	1.58927	0.58927
15	0.15	-1.03643	0.231853	0.0818528	1.54569	0.54569
16	0.16	-0.99446	0.241002	0.0810024	1.50626	0.50626
17	0.17	-0.95417	0.249967	0.0799666	1.47039	0.47039
18	0.18	-0.91537	0.258763	0.0787631	1.43757	0.43757
19	0.19	-0.87790	0.267407	0.0774075	1.40741	0.40741
20	0.20	-0.84162	0.275913	0.0759129	1.37956	0.37956
21	0.21	-0.80642	0.284291	0.0742914	1.35377	0.35377
22	0.22	-0.77219	0.292553	0.0725534	1.32979	0.32979
23	0.23	-0.73885	0.300708	0.0707082	1.30743	0.30743
24	0.24	-0.70630	0.308764	0.0687643	1.28652	0.28652
25	0.25	-0.67449	0.316729	0.0667294	1.26692	0.26692
26	0.26	-0.64335	0.324610	0.0646103	1.24850	0.24850
27	0.27	-0.61281	0.332413	0.0624134	1.23116	0.23116
28	0.28	-0.58284	0.340145	0.0601445	1.21480	0.21480
29	0.29	-0.55338	0.347809	0.0578089	1.19934	0.19934
30	0.30	-0.52440	0.355412	0.0554115	1.18471	0.18471
31	0.31	-0.49585	0.362957	0.0529570	1.17083	0.17083
32	0.32	-0.46770	0.370450	0.0504496	1.15765	0.15765
33	0.33	-0.43991	0.377893	0.0478933	1.14513	0.14513
34	0.34	-0.41246	0.385292	0.0452920	1.13321	0.13321
35	0.35	-0.38532	0.392649	0.0426490	1.12185	0.12185
36	0.36	-0.35846	0.399968	0.0399679	1.11102	0.11102
37	0.37	-0.33185	0.407252	0.0372518	1.10068	0.10068
38	0.38	-0.30548	0.414504	0.0345037	1.09080	0.09080
39	0.39	-0.27932	0.421727	0.0317266	1.08135	0.08135
40	0.40	-0.25335	0.428923	0.0289232	1.07231	0.07231
41	0.41	-0.22754	0.436096	0.0260962	1.06365	0.06365
42	0.42	-0.20189	0.443248	0.0232483	1.05535	0.05535
43	0.43	-0.17637	0.450382	0.0203819	1.04740	0.04740
44	0.44	-0.15097	0.457499	0.0174995	1.03977	0.03977
45	0.45	-0.12566	0.464603	0.0146035	1.03245	0.03245
46	0.46	-0.10043	0.471696	0.0116962	1.02543	0.02543
47	0.47	-0.07527	0.478780	0.0087800	1.01868	0.01868
48	0.48	-0.05015	0.485857	0.0058570	1.01220	0.01220
49	0.49	-0.02507	0.492930	0.0029296	1.00598	0.00598
50	0.50	0.00000	0.500000	0.0000000	1.00000	0.00000

```
MTB > let c12 = 1/ (1 + 1/(C2**2/4))   # R-sq using Z.
MTB > let c13 = .401 - .982*c1         # Regress R2 on Mdn_Ovlp
MTB > let c14 = (.5-c4)/.5             # % reduction in misclassify
MTB > Print c1 c2 c4 c12 c13 c14
```

Row	MDN_OVLP	Z	Miss	R-sq	Regr(R2)	Reduction
1	0.01	-2.32635	0.050013	0.575006	0.39118	0.899974
2	0.02	-2.05375	0.073251	0.513257	0.38136	0.853498
3	0.03	-1.88079	0.091805	0.469312	0.37154	0.816390
4	0.04	-1.75069	0.107907	0.433821	0.36172	0.784186
5	0.05	-1.64485	0.122433	0.403479	0.35190	0.755134
6	0.06	-1.55477	0.135835	0.376687	0.34208	0.728330
7	0.07	-1.47579	0.148385	0.352537	0.33226	0.703230
8	0.08	-1.40507	0.160261	0.330457	0.32244	0.679478
9	0.09	-1.34076	0.171587	0.310062	0.31262	0.656827
10	0.10	-1.28155	0.182453	0.291079	0.30280	0.635094
11	0.11	-1.22653	0.192928	0.273305	0.29298	0.614143
12	0.12	-1.17499	0.203067	0.256588	0.28316	0.593866
13	0.13	-1.12639	0.212912	0.240808	0.27334	0.574176
14	0.14	-1.08032	0.222497	0.225870	0.26352	0.555005
15	0.15	-1.03643	0.231853	0.211697	0.25370	0.536294
16	0.16	-0.99446	0.241002	0.198228	0.24388	0.517995
17	0.17	-0.95417	0.249967	0.185408	0.23406	0.500067
18	0.18	-0.91537	0.258763	0.173194	0.22424	0.482474
19	0.19	-0.87790	0.267407	0.161549	0.21442	0.465185
20	0.20	-0.84162	0.275913	0.150441	0.20460	0.448174
21	0.21	-0.80642	0.284291	0.139843	0.19478	0.431417
22	0.22	-0.77219	0.292553	0.129731	0.18496	0.414893
23	0.23	-0.73885	0.300708	0.120085	0.17514	0.398584
24	0.24	-0.70630	0.308764	0.110887	0.16532	0.382471
25	0.25	-0.67449	0.316729	0.102120	0.15550	0.366541
26	0.26	-0.64335	0.324610	0.093771	0.14568	0.350779
27	0.27	-0.61281	0.332413	0.085827	0.13586	0.335173
28	0.28	-0.58284	0.340145	0.078278	0.12604	0.319711
29	0.29	-0.55338	0.347809	0.071114	0.11622	0.304382
30	0.30	-0.52440	0.355412	0.064327	0.10640	0.289177
31	0.31	-0.49585	0.362957	0.057908	0.09658	0.274086
32	0.32	-0.46770	0.370450	0.051850	0.08676	0.259101
33	0.33	-0.43991	0.377893	0.046148	0.07694	0.244213
34	0.34	-0.41246	0.385292	0.040796	0.06712	0.229416
35	0.35	-0.38532	0.392649	0.035790	0.05730	0.214702
36	0.36	-0.35846	0.399968	0.031123	0.04748	0.200064
37	0.37	-0.33185	0.407252	0.026794	0.03766	0.185496
38	0.38	-0.30548	0.414504	0.022798	0.02784	0.170993
39	0.39	-0.27932	0.421727	0.019132	0.01802	0.156547
40	0.40	-0.25335	0.428923	0.015793	0.00820	0.142154
41	0.41	-0.22754	0.436096	0.012779	-0.00162	0.127808
42	0.42	-0.20189	0.443248	0.010087	-0.01144	0.113503
43	0.43	-0.17637	0.450382	0.007717	-0.02126	0.099236
44	0.44	-0.15097	0.457499	0.005666	-0.03108	0.085001
45	0.45	-0.12566	0.464603	0.003932	-0.04090	0.070793
46	0.46	-0.10043	0.471696	0.002515	-0.05072	0.056608
47	0.47	-0.07527	0.478780	0.001414	-0.06054	0.042440
48	0.48	-0.05015	0.485857	0.000628	-0.07036	0.028286
49	0.49	-0.02507	0.492930	0.000157	-0.08018	0.014141
50	0.50	0.00000	0.500000	0.000000	-0.09000	0.000000

```
MTB > corr c1-c4 c12-c14

       MDN_OVLP        Z       Zp     Miss     R-sq   Regr(R2)
Z         0.966
Zp        0.966    1.000
Miss      0.994    0.988    0.988
R-sq     -0.932   -0.991   -0.991   -0.966
Regr(R2) -1.000   -0.966   -0.966   -0.994    0.932
Reductio -0.994   -0.988   -0.988   -1.000    0.966    0.994
```

Suppose that a distribution is a mixture that can be formed from two different pairs of distributions. Suppose the distribution of heights can be formed from the distribution of heights for males and females or from the distribution of heights of smokers and non-smokers.

The relation between the parameters of a mixture and those of the two parts is given by:

$$N = n_1 + n_2 \quad \text{Eq 1.}$$

$$\mu_0 = (n_1 \mu_1 + n_2 \mu_2)/ N \quad \text{Eq 2.}$$

$$\sigma_0^2 = \{n_1 [s_1^2 + (\mu_1 - \mu_0)^2] / N\} + \{n_2 [s_2^2 + (\mu_2 - \mu_0)^2] / N\} \quad \text{Eq 3.}$$

$$\{n_1 s_1^2 + n_2 s_2^2 + n_1 (\mu_1 - \mu_0)^2 + n_2 (\mu_2 - \mu_0)^2\} / N \quad \text{Eq 4.}$$

$$\{n_1 s_1^2 + n_2 s_2^2 + n_1 (\mu_1 - \mu_0)^2 + n_2 (\mu_2 - \mu_0)^2\} / N \quad \text{Eq 5.}$$

$$\{n_1 s_1^2 + n_2 s_2^2 + n_1 (\mu_1 - \mu_0)^2 + n_2 (\mu_2 - \mu_0)^2\} / N \quad \text{Eq 6.}$$

$$n_1 (\mu_1 - \mu_0)^2 + n_2 (\mu_2 - \mu_0)^2 = n_1 (\mu_1^2 - 2\mu_0\mu_1 + \mu_0^2) + n_2 (\mu_2^2 - 2\mu_0\mu_2 + \mu_0^2) \quad \text{Eq 7}$$

$$= n_1 \mu_1^2 - 2 n_1 \mu_0\mu_1 + n_1 \mu_0^2 + n_2 \mu_2^2 - 2 n_2 \mu_0\mu_2 + n_2 \mu_0^2 \quad \text{Eq 8}$$

$$= n_1 \mu_1^2 + n_2 \mu_2^2 - 2 \mu_0 (n_1 \mu_1 + n_2 \mu_2) + N \mu_0^2 \quad \text{Eq 9}$$

$$= n_1 \mu_1^2 + n_2 \mu_2^2 - 2 N \mu_0^2 + N \mu_0^2 \quad \text{Eq 10}$$

$$= n_1 \mu_1^2 + n_2 \mu_2^2 - N \mu_0^2 \quad \text{Eq 11}$$

$$\sigma_0^2 = \{[n_1 s_1^2 + n_2 s_2^2] + [n_1 \mu_1^2 + n_2 \mu_2^2 - N \mu_0^2]\} / N \quad \text{Eq 12}$$

Suppose n1 = n2 = N/2

$$\sigma_0^2 = \{[s_1^2 + s_2^2] + [\mu_1^2 + \mu_2^2 - 2N \mu_0^2]\} / 2N \quad \text{Eq 13}$$

$$= \{[s_1^2 + s_2^2] + [(\mu_2 - \mu_1)^2 + 2\mu_2 \mu_1 - 2N \mu_0^2]\} / 2N \quad \text{Eq 14}$$

Consider two pairs (A and B) each having two parts (1 and 2):

Pair A: Separation = $\Delta\mu_A = \mu_{A2} - \mu_{A1}$ Eq 13

Count: $n_{A2} = N - n_{A1}$ Eq 14

$$n_{A2} s_{A2}^2 = N\sigma_0^2 – [n_{A1} s_{A1}^2] - [n_{A1} \mu_{A1}^2 + n_{A2} \mu_{A2}^2 - N \mu_0^2] \quad \text{Eq 13}$$

$$n_{A2} s_{A2}^2 = N\sigma_0^2 - [n_{A1} s_{A1}^2] - [n_{A1} \mu_{A1}^2 + n_{A2} \mu_{A2}^2 - N \mu_0^2] \quad \text{Eq 14}$$

$$n_{A2} \mu_{A2} = N \mu_0 - n_{A1} \mu_{A1} \quad \text{Eq 15}$$

$$n_{A1} \mu_{A1}^2 + n_{A2} \mu_{A2}^2 - N \mu_0^2$$

$$= n_{A1} \mu_{A1}^2 + \{[N \mu_0 - n_{A1} \mu_{A1}]^2 / \}/ n_{A2} - N \mu_0^2 \quad \text{Eq 16}$$

$$= n_{A1} \mu_{A1}^2 + [N \mu_0 - n_{A1} \mu_{A1}]^2 - N \mu_0^2 \quad \text{Eq 17}$$