WHAT DOES "100% JUICE" MEAN?
EXPLORING ADULT LEARNERS' INFORMAL KNOWLEDGE OF PERCENT

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Abstract

This report examines adult students' informal knowledge of percent and its relationship to their formal computational skills. Sixty adults studying in adult education programs were interviewed to ascertain (a) their ideas of the meanings of three benchmark percents, 100%, 50%, and 25%, as they appear in advertising and media contexts; (b) their ability to use these numbers in everyday mental math tasks; and (c) their visual representations of these quantities. Students also completed school-like, computational percent exercises. Students' responses were examined to determine the nature of their informal knowledge and skills; a number of patterns of informal knowledge and formal skills were identified. The range and fragility of student responses and the diversity of existing knowledge gaps suggest the need to broaden the content of percent instruction beyond computation. Mathematics assessments should be expanded to include performance on real-world tasks and should involve probing of responses to explore depth of understanding.

INTRODUCTION

Percents are central to many aspects of adults' lives; knowledge of percents is required for effective understanding of and performance in numerous real-world situations, such as managing personal finances (e.g., understanding interest), handling functional tasks (e.g., understanding discounts), and dealing with work-related tasks (e.g., generating indicators of product quality). Some conceptual understanding of percents is also essential for comprehension of messages in the media, such as statistical information about economic or social trends (e.g., changing tax rates or pollution levels).

Given the importance of the concept of percent, it is surprising that little is known about children's learning and difficulties with percent, and that apparently no research has examined adults' knowledge of percent. Research has focused on exploring children's acquisition of rational numbers and proportional ideas (Behr, Harel, Post, & Lesh, 1992; Mack, 1990; Wearne & Hiebert, 1988) rather than their understanding of percent. This reflects the view taken by many teachers and textbooks (in both K-12 and adult education) that the study of percent is only an extension of the study of fractions and decimals. This approach leads to an emphasis on the mechanics of percent-related calculations or converting between percents, fractions, and decimals, rather than on conceptual understanding.

The few studies that have examined children's knowledge of percents looked only at specific cognitive processes in limited contexts: Venezky and Bregar (1988) focused on solving traditional word problem tasks, Streefland and van den Heuvel-Panhuizen (1992) explored fifth graders' informal knowledge displayed in response to four daily-life stories, and Joram, Raghavan, and Resnick (1992) investigated students' interpretation of percents found in text. A recent study by Lembke and Reys (1994) employed a broader research framework, and examined the performance of students in grades 5, 7, 9, and 11 on decontextualized and situational percent problems. Among other findings, these researchers reported that older students used a narrow range of solution approaches, usually based on procedures learned in school. In contrast, younger students used a more diverse range of strategies and showed greater reliance on benchmark percents, such as 50% and 25%. Lembke and Reys also reported that students' conceptual understanding and ability to solve percent problems increased with age, but that even at the eleventh grade, middle-ability students could not solve one fourth of all problems. They concluded that "Formal instruction in the application of percent tends to make the students' concepts of percent less intuitive and more rule-driven, actually narrowing rather than expanding the strategies and the computational methods students use when working with percents" (p. 256).

These and other findings, such as results from the National Assessment of Educational Progress (Mullis, Dossey, Owen, & Phillips, 1991), imply that many students are likely to leave school with an incomplete conceptual understanding and computational knowledge of percent, as well as with insufficient preparation for using or understanding the use of percents in the real world. This assumption is bolstered by results from the recent National Adult Literacy Survey (Kirsch, Jungeblut, Jenkins, & Kolstad, 1993), which showed that between a quarter and a half of the adult population in the United States have difficulty dealing with many functional tasks involving mathematical elements, including numerous tasks with percents.

Adults and Percents

This study was designed as a preliminary exploration of adults' knowledge of percents, with a focus on adults who are returning to an educational setting to study math, such as in a literacy, basic skills, or an employment-preparation program. We will see that this population is significant in size.

Based on a survey of a representative sample of 350 adult education programs in the United States, Gal and Schuh (1994) estimate that approximately 80% of the almost four million adults presently studying in state-administered adult education programs each year receive some math education; roughly half of those who study math are classified at the adult basic education level (ABE, usually taken as equivalent to grades 1-8) and the remainder at the adult secondary education level (ASE, mostly including those preparing for high-school equivalency exams such as the GED). While adult education programs in America may teach or re-teach percents to millions of adult students, we have found no research on adults' understanding of and difficulties with percents. (A search of the ERIC and PSYCHLIT databases did not uncover any report of research involving adults' understanding of percent. The National Adult Literacy Survey (Kirsch et al., 1993) included questions involving percents but thus far, published reports have not described adults' responses or solution strategies for these particular tasks.) Such knowledge is needed to design effective teaching and to inform training of teachers and curriculum development.

In designing the present study, it seemed important to attend not only to the formal skills and knowledge that adults may already have, but also to the informal knowledge and skills about percents that they bring to their studies. Informal knowledge is likely to be shaped by adults' particular life circumstances and experiences after formal schooling ended. We expected that most adults in the United States, including those who did not complete high school, frequently come into some contact with percents in their daily lives, and that adults develop some intuitions or ideas about percents, even if they have not formally studied (or fully understood) percents while in school.

One important aspect of adults' informal experience with percents, which may be somewhat different from the informal experiences of younger people, pertains to the greater exposure that adults can be expected to have to "interpretive situations" (Gal, 1993; Kirsch et al., 1993), such as percents embedded in media-based messages, which require no calculations but rather comprehension and critical analysis. In addition, adults, more so than children, may come across situations requiring quick "ballpark" estimation such as when shopping or dealing with discounts. In such contexts, effective functioning does not necessarily require people to have strong computational skills, but rather a general, perhaps intuitive, understanding of the percent system, as well as "number sense" and mental math skills (Sowder, 1988).

The informal mathematical conceptions and intuitions that adults (and children) carry with them may vary in their degree of "correctness" (Fischbein, Deri, Nello, & Marino, 1985; Leinhardt, 1988; Mack, 1990; Nunes, Schliemann, & Carraher, 1993; Riley, Greeno, & Heller, 1983). Information about such informal or prior knowledge that adults bring with them to their studies of percents is essential for the design of effective instruction that builds on students' strengths and tries to ameliorate knowledge gaps or misunderstandings.

RESEARCH APPROACH AND QUESTIONS

We chose to explore adults' responses to a variety of tasks involving percents, including tasks that people generally encounter only in school as well as tasks that adults encounter in everyday situations, such as shopping and interpreting statements with percents. The subjects in this study are adults who returned to school because their mathematical (and other basic) skills have been deemed inadequate by themselves or others. These adults may have informally developed ideas and intuitions about percents (and many other mathematical topics) to enable them to function in everyday situations. Our goal was to document if and how adults modify or invent percent-related ideas in ways that are meaningful and useful to them.

We focus in this paper on students' understanding of three specific percent values: 100%, 50%, and 25%. Knowledge of the percent system requires an appreciation of 100% as the basis for the system. From an informal survey of advertisements in the media and in stores, we found 50% and 25% to be among the most commonly used percents. In addition, these two percents are mathematically related and easily converted into familiar fractions and decimals. Thus, we expected that adult students would likely have been exposed to these two percents and may have developed personally useful informal knowledge and strategies for dealing with them. Lembke and Reys (1994) identified these percents as "benchmark percents" and we will use that terminology as well.

The specific questions addressed in this study are as follows:

- What knowledge do adult students' have of 100% and its role as the basis of the percent system?
- How do adult students make sense of and solve problems involving the benchmark percents 50% and 25%?
- How are informal knowledge and skills related to formal computational skills?
- Are the standardized tests that most adult literacy programs use to inform placement and advancement decisions sensitive to the level of adults' informal and formal knowledge about percent?

METHOD

SUBJECTS

This study involved 60 adults from seven adult education classes, three serving inner city and four serving suburban populations. The sample included 3 men and 57 women, ranging in age from 18 to 53 years, with a mean age of 27.5 years. Only a small number of men participated in the study because five of the programs were either designated by funding agencies as "women's programs" or were aimed at reducing the dependence of single parents (who are usually women) on public assistance. Fifty-three percent of the students were African American, 14% were Hispanic, and 33% Caucasian. The students had been studying math (and in most cases, also other subjects) in their classes for between 2 weeks and 3 months but had not yet engaged in learning percent. All students in each designated class were given the opportunity to participate in the study. All or almost all students in each class expressed curiosity and interest in participation.

The stated purposes of the seven educational programs were different, but the goals regarding mathematics were the same for all programs—to review the traditional mathematics curriculum and improve the computational skills of the students. The programs aim to prepare students for successful performance on standardized tests, primarily those used to screen applicants for employment training or the GED test.

The vast majority of the students completed between 9 and 12 years of school (mean=10.6 years). Three students completed between 6 and 8 years of school and 3 others took courses at a community college for either one or two years. The programs provided students' scores on the math subtest of standardized tests used at the intake stage; five programs used the TABE, the Tests of Adult Basic Education; and two used the ABLE, the Adult Basic Literacy Education test (see Sticht, 1990, for more details on these tests). When expressed in grade-equivalent units (a common practice in adult education programs), 15 students (25% of the sample) were classified as having scores in the grade range 4.9-6.9, 24 students (40%) in the 7.0-8.9 range, and 15 (25%) scored in grade range 9.0 and above (scores were unavailable for 6 students). Thus, students' achievement on standardized tests identified them as having lower computational skills than would be expected on the basis of their prior education. In the analyses reported below, the above-defined grade-level equivalents, rather than the number of years of schooling completed, were used to describe groups of students. The use of three grade-level groups provides better discrimination than the use of prior years of schooling for two reasons: (a) 43% of the students in the sample attended school for either 11 or 12 years yet are spread over all three grade-level groups; they comprise half of the highest grade-level group, half of the middle-scoring group, and one fourth of the lowest scoring group; and (b) there is no way to describe or evaluate the content of former schooling for these students.

PROCEDURE

Overall, students were presented with four different types of tasks involving percent: *explanatory tasks*, *shopping tasks*, *visual tasks*, and *computation tasks*. The first three types of tasks were presented as part of an individual, semistructured interview, which lasted approximately 30 minutes. Since students participate in adult education programs on a voluntary basis and often attend only for a limited time, overall assessment time was kept to a minimum. To ensure that reading difficulties would not affect students' responses, the interviewer read pertinent parts of the stimulus materials aloud. Students' statements were recorded in writing and also audiotaped. About three weeks after the interviews, students at each site completed a brief written assessment of computational skills as part of their regular classwork.

EXPLANATORY TASKS

The first part of the interview was designed to provide information on adult students' ideas about five interrelated facets of the meaning of 100% and the role of 100% in the percent system. We identified these facets, described below, as important in conceptualizing the structure of the percent system and as useful in comprehension of statements about percents as they appear in everyday situations. The tasks used to explore these five facets are listed in Table 1.

- Awareness that percents are expressed on a scale from 0-100; recognition of this 0-100 scale and its ordinal nature is the foundation upon which people can base simple interpretations of percent-based statements even if they are unable to compute with percents. (Percents larger than 100 or smaller than 1 were not addressed in this study, as they are much less frequently encountered.)
- Knowledge that the percentages of the parts of a whole must add up to 100%.
- Recognition that a percentage of a whole can be represented visually in a way that conveys a sense of its proportional nature or relative size.
- Identification of 100% as meaning "whole" or "all."
- Appreciation of the invariability of 100% as the reference point for percent (i.e., that the use of 100 [and not any other number] is an accepted convention in describing proportional amounts).

Table 1

Interview Tasks Used to Assess Knowledge of 100% as the Basis of the Percent System

Facet	Stimulus presented	Questions asked
1. Percents lie on an ordinal scale ranging from 0 to 100.	Circled line from newspaper article stating that a new blood test "detects cancer correctly in 90% of cases."	Do you think this is a good test? Why? What does the 90% tell you?
2. Percentages of the parts of a "whole" sum to 100%.	Printed statement purportedly from a magazine stating "In 1970, 15% of all American children were living in single parent homes."	Can you tell what percent of the children were <u>not</u> living in single parent homes?
3. Visual representation of a percent as a proportional part of a whole.	Same as #2 above.	If this circle represents all the children in the United States during 1970, about how big a slice of it would be 15%?
4. 100% means "whole" or "all."	Bottle of apple juice that says "100% Juice."	What does "100% Juice" mean?
5. Use of 100% as the invariable reference point for percents.	Follow up question when subject mentioned 100% during the interview.	Why did you use 100%? Could you have used something else?

These five facets represent a somewhat different approach than that taken by Lembke and Reys (1994) and Allinger and Payne (1986), whose research focused on students' knowledge of percent in an instructional context and thus emphasized the mathematical foundations of percent. The present research focused more on the knowledge that adults may have developed and bring to a new educational setting, and less on computational, school-based knowledge, which is less relevant in the case of adults who have been out of school for some time.

SHOPPING TASKS

The second part of the interview focused on students' reasoning about specific percents that they may encounter in an everyday shopping context. This context was chosen on the assumption that it would be familiar to all students, and that it can bring to the surface informal knowledge that students may have developed or otherwise incorporated into nonschool contexts. Students were shown advertising flyers containing percent statements such as "Sale, 50% off" and were asked to explain their ideas of the meaning of the highlighted percents. Questions that elicited students' ideas of the meaning of 50% and 25% are shown in Table 2 under the column titled "Meaning of the percent."

Table 2

Tasks With Benchmark Percents

	Task type			
Percent	**Meaning of the percent**	**Shopping tasks**	**Visual tasks**	**Written computational tasks**
50%	Here it says, "50% off." Can you explain what 50% means?	How much would you pay for $10 pants on sale at 50% off?		What is 50% of 10?
	Why is 50% equal to ...?		The 10 people in the circle will work on math during the class: (10 out of 20 =?%)	10 is what % of 20?
25%	Here it says "25% off." Can you explain what 25% means?	How much would you pay for an $80 coat on sale at 25% off?		
	Why is 25% equal to ...?		25% of the 20 students want to take the GED test: (25% of 20=)	What is 25% of 20?

Next, after students described their shopping practices, they were asked to imagine being in a store and wanting to know, before approaching the cashier, how much items on sale would cost. Students were shown drawings of priced items and discount figures in percent (e.g., a drawing of pants with a price tag stating "$10" and a sign saying, "Pants 50% off"); they were asked to find the discounted prices mentally, or to approximate the answers if they were otherwise unable to come to an answer. No written computations were allowed in this part since students would be unlikely to use paper and pencil while shopping. Students were asked to think aloud during the process and to explain their reasoning. Questions that used 50% and 25% in shopping contexts are shown in Table 2 under the column titled "Shopping tasks."

VISUAL TASKS

Visual tasks were used to explore additional aspects of students' reasoning about percent. These tasks were designed to enable students to reason about percent in a context that was different from the everyday, familiar encounter (shopping) yet was not a school-20 stick figures (see Figure 1) and were asked to assume that the figures represented 20 students in a class. In some tasks, students were given a percent value and were asked to circle the corresponding proportion of stick figures. For example, students were presented with the diagram of 20 stick figures and the statement "25% of the students in this class are planning to take the GED test" and were asked to circle the number of people corresponding to 25%. In other tasks, students were shown the diagram with a number of stick figures circled and were asked what percent of the 20 figures was circled. The questions about 50% and 25% used in this part are shown in Table 2 under the column titled "Visual tasks." Due to constraints on interview length, mathematically equivalent questions were not asked in the shopping and visual formats.

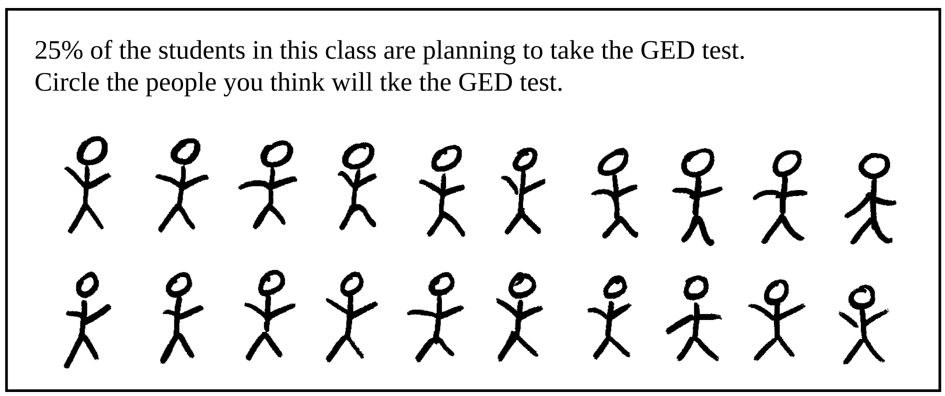

Figure 1. Sample Visual Percent Task

WRITTEN COMPUTATIONAL TASKS

The written assessment included three arithmetic items similar in mathematical structure to items in the interview. Two of the items used 50% and one used 25%. These items are shown in Table 2, under the column titled "Written computational tasks," and are shown horizontally across from the mathematically equivalent shopping or visual question.

CODING

Responses to interview tasks that involved explanations of meaning rather than numerical values were categorized as "appropriate" or "inappropriate"; the criteria for this identification differed for each question and are explained below. Numerical responses were coded as "correct," "in the ballpark" (within 10% above or below the correct response for the shopping questions since such an estimate might well be useful in that context), or "incorrect," and the solution strategies used were described and grouped together into categories. Since students completed the written computational tasks in their class settings, no observations or explanatory information were available about their responses, and the responses were only coded as arithmetically correct or incorrect.

RESULTS

KNOWLEDGE OF 100% AS THE BASIS OF THE PERCENT SYSTEM

This section examines students' knowledge of the five basic facets about the percent system discussed above, all of which revolve around the notion of using 100% as the basis for thinking about percent. While each of the five questions presented to students was designed to elicit their knowledge about a different facet, each question very often elicited responses related to other facets as well. As students' ideas about these five facets are all interrelated, a discussion of students' knowledge should take into account all answers rather than single responses in isolation.

IDENTIFYING PERCENT AS LYING ON A 0-100 SCALE

Students were asked to interpret the statement, "This new test detects cancer correctly in 90% of the cases," which appeared in a newspaper clipping shown to them. Almost all students (93%) demonstrated knowledge (either on their own or after probing) that 90% should be evaluated in relation to 100%, where 100% would be a perfect detection rate.

In the sample responses quoted below, two students' conclusions about the acceptability of a 90% success rate were different, yet both students indicated that the relative magnitude of a percent value is judged based on its proximity to 100%, and were thus scored as appropriate.

Steve: It's a good test. Ninety percent is almost all cases. It's close to 100%.
Laura: I wouldn't depend on the test, 90% is not good enough for me.
Interviewer: What would be perfect?
Laura: 100%.

The remaining 7% of the students provided inappropriate responses, which did not mention 100% as a reference point for evaluating 90%, but rather appeared to interpret 90 in absolute terms.

Sandy: It's good because 90% is a high percentage.
Interviewer: What would be the highest?
Sandy: I don't know.
Interviewer: How about 150%?
Sandy: That's higher, that would be better.
Interviewer: What would be perfect?
Sandy: I don't know.

RECOGNIZING THAT PERCENTAGES OF THE PARTS OF A "WHOLE" ADD UP TO 100%

Students were presented with a statement based on data from the 1970 Census, "In 1970, 15% of all American children lived in single parent homes," and were asked if they could tell what percent of children were *not* living in single parent homes. Most students (83%) responded immediately with "85%" and indicated that they expected the two percents to complement each other and sum to 100%.

Of the 10 students (17%) who did not demonstrate understanding that the sum of the percentages should be 100%, 6 stated that they could not tell what percent of the children did not live in single parent homes because they did not know the total number of people, suggesting they were having difficulty taking advantage of the proportional nature of percent.

The other four students either guessed at answers or performed inappropriate computations reflecting a "number-grabbing" strategy, in which they attempted to compute with whatever numbers were available, even including the year that had been mentioned as part of the question.

Susan: The rest of them, 15 times 1970. You don't have the other number, you don't have the all, you don't have the other people. So put 1970 over 15, you have to estimate. Divide 15 into 1970. Whatever is left over, I think 11%.

Visual Representation of a Percent as a Part of a Whole

In connection with the statement, "In 1970, 15% of all American children lived in single parent homes," students were presented with a circle (pie) containing densely drawn dots, said to represent all the American children in 1970. Students were asked to shade a slice of the circle representing the 15% of the children who lived in single parent homes.

About three quarters (72%) of the students shaded a section somewhere between 10% and 20% of the circle. Most of these students divided the circle into four quarters and shaded in approximately half of one of the quarters. Some students divided the circle into 10 parts, identified each part as 10%, and then shaded one and one half parts. Others mentally estimated and shaded approximately 15% without first partitioning the circle into any series of equal parts, but were able to explain why their estimates were reasonable. All of these responses were scored as "appropriate" since they implied that the entire circle represented 100% (and thus that 15% was a proportionally small section).

Of the seventeen students (28% of sample) whose responses were scored as inappropriate, eight divided the circle into 15 equal parts. These students implied that 15% meant one fifteenth, and they did not recognize the idea of percent as a proportion based on 100. Several students indicated that 15% is "a little bit" (which shows rudimentary understanding of the ordinal 0-100 scale) but were unable to explain why this is so. The remaining students in this group shaded different portions of the circle as 15%, and provided no indication, even after probing, that for them the entire circle represented 100%.

Knowing That 100% Means "Whole" or "All"

To provide additional information about students' understanding of 100% as the basis of the percent system, students were asked to interpret the statement "100% juice," printed on a bottle of apple juice, which was shown during the interview. In this context, 83% of the students stated that to them, 100% means "all" or "wholly" juice.

Anna: It's all juice, all apple juice.
Interviewer: Can there be anything else in there too?
Anna: No, there shouldn't be.

Several of the students who stated that "100%" means "all" conveyed a sense of a contradiction between the mathematical meaning of 100% (i.e., that 100% *should* mean "all") and its actual meaning in the apple juice context (i.e., showing awareness of the real-world use or misuse of percent, particularly related to advertising).

April: It's basically natural, all juice, it doesn't have excess water and other things, like when they say 10% juice. Nothing that makes it taste of apple other than the apple itself.
Interviewer: So there's nothing else but juice?
April: I guess, vitamins, like vitamin A. When it says 100% juice, it's just that kind of juice. I wouldn't think there's nothing else but juice, but mainly juice. If it's 100%, it may not be 100% all juice but mainly all juice. For example, decaf coffee has some caffeine.

In contrast, 17% of the students responded that 100% either means "the majority" of something, rather than "all" (first quote below) or else represents an absolute quantity or amount that is not related to proportionality (second quote below).

Katrina: More juice than whatever they use to make the juice. More of it is natural; it contains less sugar than the rest of the drinks. . . . More fruit juice in this bottle.
Interviewer: Can there be other ingredients in there too?
Katrina: Yes.

Tara: You're going to be drinking 100% juice. There may be other ingredients, but you would want to have at least that much, 100%. I would also look at the other ingredients, sugar, sodium.
Interviewer: Do you mean the juice could be higher than 100% juice?
Tara: It could be. If it said 100%, I would think it would be at least 100% and I would buy it.
Interviewer: So it could say a higher number than 100%?
Tara: Yes. I've never seen one. If I did I would assume it was more than 100%, if it added another type of apple, two different types of apples.

USE OF 100% AS THE REFERENCE POINT FOR PERCENTS

To further explore students' knowledge of the special and invariant use of 100% as the basis for the percent system, students were asked to explain why they referred to 100% in an earlier question (e.g., when deciding that 85% of the children were *not* living in single parent homes), and whether any other percent could have been used instead. Only in responding to such a direct probe were the students forced to justify their use of 100% as an organizing principle of the percent system, rather than having this idea inferred from their answers to other questions.

Thirty-one of the 60 students (52%) provided appropriate explanations for their use of 100% by suggesting that 100% means a "whole" or "all of something."

Interviewer: Why did you use 100%?
Lois: A whole is 100%.
Interviewer: Is it always 100%?
Lois: More or less. It depends on if you're dealing with numbers. Well, yes, it's always 100% because a whole is 100%.
Interviewer: Could you have used a number other than 100%?
Lois: No.

Some of those who gave appropriate responses appeared to be crystallizing or rethinking their ideas about percents even as they constructed their responses, and they demonstrated that their understanding of 100% as the basis for percent is still evolving. Others tied their thoughts about 100% to their life experiences (e.g., with money) rather than to an abstract principle or a mathematical convention.

Interviewer: Why did you use 100% [to compute the number of children not living in single parent homes]?

Tracey: I always do it by 100, I could do it by 200 but you would have to double everything. I didn't try it by 200, so I don't know if it would work.

Interviewer: So you could have used 200%?

Tracey: I don't know. You probably could, but it would be more work. 100% of 200 would still be 100%, 100% is everything, all of it. I guess you can't do it by 200%, maybe some people could do it.

Interviewer: Why did you use 100?

Dorothy: It all depends on how you're breaking it down. You can use any number for a whole: fifty fiftieths, four fourths.

Interviewer: And when you are dealing with percent?

Dorothy: It would have to be over 100, 200% could be a whole, 250% couldn't be a whole because that breaks the rhythm.

Interviewer: So which numbers can be a whole?

Dorothy: Zeros: 100, 200, 300.

Interviewer: As high as you want?

Dorothy: All depends on what type of money you're dealing with. Got 10 million dollars (pause). No keep it at 100%, forget the 200%, etc. 100% is a whole.

The other 29 students (48% of the sample) provided one of several inappropriate responses. Fifteen of these students (25% of the sample) were confident that 100% was the right number to use but were unable to articulate reasons why that was so other than "Percent is always 100," "It seems right," or "It's in my head." Five students were unsure why they chose 100%; they indicated that other numbers could have been used instead, but were unsure of what such numbers could be. Two students understood 100% as 100 individuals or cases, without acknowledging any proportional nature of percent.

Interviewer: Would you always use 100% [to evaluate 90% in "This new test detects cancer correctly in 90% of the cases"]?

Theresa: Yes. In a way, you don't know. It all depends on how many cases they used. 90% is good out of 100% of the people. If you have 250, 90% is not good. It's not half of 250 people. 125 would be half.

Interviewer: Do you mean if there were 250 people, then 125% would be half?

Theresa: Yes.

Four students explained that they used 100% because it is "the highest percent can go," and invoked their experience of not encountering a percent greater than 100 rather than a conceptual understanding of how the percent system is structured.

Interviewer: Why did you say 100%?

Arlene: That's as high as I think you can go with percentage. I never heard 200% off, 220% off. I never seen any number that's more than 100%, not in stores.

Interviewer: So 100% is the highest?

Arlene: More or less. Unless you're taking bigger numbers. If you want a percent off of $100,000, (pause) doesn't make sense. No, 100% is the highest, no matter what amount of money.

Finally, three students explained that 100% was used because it is a "round number"; they seemed to equate (and confuse) round numbers with rounding of numbers, and did not provide any particular reason why 100 was used in dealing with percents.

Interviewer: Why did you use 100%?
Tamika: It's a round number, a whole number. I was taught to use it, you have to round something off and the number was always 100. . . . The nearest 10; 100; 1000; 100,000—always rounded off. They're supposed to come out even.
Interviewer: So why did you use 100%?
Tamika: It was easier.
Interviewer: Could you have picked something else?
Tamika: I could have, 1000, or 1,000,000.
Interviewer: Numbers with zeros?
Tamika: They don't necessarily have to be zeros. You can round off numbers in between too. Even the remainder you can round off.

RESPONSE PATTERNS AND EDUCATIONAL ACHIEVEMENT

Tables 3 and 4 provide two different summary views of students' knowledge of 100% as the basis of the percent system (as measured by the five questions described above).

Table 3

Percentages of Appropriate Responses for Each Facet of Role of 100% as the Basis of the Percent System

Facet	Grade level			Unclassified	Total
	6th and below	7th-8th	9th and above		
	n=15	n=24	n=15	n=6	n=60
Percents lie on a 0-100 scale	87%	96%	93%	100%	93%
Percentages of the parts of a "whole" sum to 100%	80%	79%	100%	67%	83%
Visual representation of percent as proportional part of whole	67%	71%	67%	100%	72%
100% means "whole" or "all"	80%	83%	87%	83%	83%
Use of 100% as the reference point for percents	40%	38%	80%	67%	52%

In Table 3, the percentage of students who responded appropriately to each of the five questions is shown. Table 4 shows cumulative data across these five questions. A discussion of performance in relation to educational achievement continues later in the section titled, "Educational achievement and performance patterns."

Almost all students showed at least some familiarity with the percent system, as indicated by the finding that most responded appropriately to at least three of the five questions (see Table 4). Those who were able to justify their use of 100% (the fifth question listed in Table 3) were also likely to be those who responded correctly to the other four questions, indicating a familiarity with several basic facets of the percent system. Other than that, there was no discernible pattern of errors; different facets were poorly understood by different individuals, suggesting that many adults may have isolated intuitions about percent that are not part of an elaborated knowledge structure.

Table 4

Percentage* of Students Within Grade Levels by Total Number of Appropriate Responses to Questions About 100% as the Basis of the Percent System

	Grade level				
Total number of appropriate responses	6th and below n=15	7th-8th n=24	9th and above n=15	Unclassified n=6	Total n=60
0	7%	0%	0%	0%	2%
1	0%	4%	0%	0%	2%
2	20%	4%	7%	0%	8%
3	13%	29%	13%	17%	20%
4	27%	46%	27%	50%	37%
5	33%	17%	53%	33%	32%

***Note:** Percentages are calculated within each grade level. Columns may not sum to 100% due to rounding.

PERFORMANCE ON TASKS INVOLVING 50%

Certain percents, such as 50%, appear frequently in everyday life and are also useful as benchmark percents for estimation or mental math tasks. An understanding of the meaning of 50% and an ability to apply that understanding flexibly are foundation blocks of a student's ability to deal with a variety of percent-laden situations. To explore students' knowledge and skills relating to 50%, five tasks were presented: interpretation of a statement involving 50% (no computation required), mental computation with 50%, identification of a given portion of a visual array as 50%, and "paper and pencil" computation of two school-like arithmetic tasks mathematically equivalent to the earlier tasks (see Table 2).

The first two questions were couched in the context of department store shopping. All students stated that they had frequently seen the type of local department store advertising flyer that they were being shown. The visualization question was accompanied by a diagram (as shown in Figure 1) of 20 stick figures that were described as representing 20 people in an adult education class. All students indicated that they understood the diagram. The diagram of stick figures was used rather than shaded areas of circles or squares (which are common in research on rational numbers) because the stick figures were less abstract and could more obviously represent a familiar, real situation to the students. The percentages of correct responses to the 50% questions for students grouped by grade level are shown in Table 5.

Table 5

Percentages of Appropriate or Correct Responses to Questions About 50%

Question type	Question asked	Grade level: 6th and below	7th-8th	9th and above	Unclassified	Total
		n=15	n=24	n=15	n=6	n=60
Meaning	What does 50% mean?	100%	100%	100%	100%	100%
Shopping	What is 50% off $10?	100%	92%	93%	83%	93%
Written computation	What is 50% of 10?	67%	79%	87%	67%	77%
Visual	10 is what % of 20?	73%	88%	100%	83%	87%
Written computation	10 is what % of 20?	27%	33%	60%	17%	37%

EXPLAINING THE MEANING OF 50%

Students were presented with an advertising flyer stating “50% off” and asked what 50% means in this context. All 60 students explained 50% as “one half.” When asked to justify why 50% is the same as one half, 46% of the students explained that 50% is one half of 100% or that 50 is one half of 100. Forty-one percent of the students used a money metaphor to explain that 50% is one half: “50 cents is one half of a dollar” or “$50 is one half of $100.” The remaining 13% were confident that 50% meant one half but were unable to state any reason or justification for this other than “I just think it is.”

MENTAL MATH AND WRITTEN COMPUTATION WITH 50%

Almost all students (93%) were able to state correctly how much would have to be paid if a $10 pair of pants were on sale for “50% off.” Practically all of them first converted 50% to its fractional equivalent, one half; stated that half of $10 is $5; and then took $5 from $10 to yield their answer. The remaining 4 students (5%) were unable to generate any ideas on how they could arrive at an answer other than by guessing. In contrast, 77% of all students responded correctly to the mathematically equivalent question on the written assessment (50% x 10=?).

VISUAL REPRESENTATION AND WRITTEN COMPUTATION WITH 50%

When asked what percent of the array of 20 stick figures were enclosed in a circle containing 10 figures, 87% of the students responded correctly, and all but one explained that because one half of the stick figures were inside the circle, the circle contained 50% of the figures. One person established that each figure represented 5% through trial and error guessing, and then counted by 5s to 50.

The 13% who did not answer correctly either named the number of figures within the circle as the percent or guessed at an answer without being able to explain a rationale for the guess.

In contrast, only 37% of all 60 students responded correctly to a mathematically equivalent computational question, "10 is what % of 20?" The large gap in solution rates between the written and visual forms of the question was apparent for students in all grade-level groups. Of the 15 students in the highest grade-level group, 100% responded correctly to the visual task but only 60% solved the equivalent written problem; even larger percentage differentials were found for the middle and lowest grade-level groups.

Response Patterns Across Tasks Involving 50%

All students answered at least one question appropriately. Twenty students (33%) responded appropriately to all five questions using 50%.

Thirty students (50%) understood the meaning of 50% and were able to apply their understanding of 50% in a shopping context and in a visual task, yet they failed to solve either one or two mathematically equivalent written computational problems ("What is 50% of 10?" and "10 is what % of 20?"). Nine students failed on both computational problems, and 21 students failed to solve only the second question. These two forms of percent problems ("find the percentage quantity" vs. "find the percent") are often presented in school settings as completely different tasks requiring different algorithms for solution rather than as complementary forms of the same concept. Possibly, the students knew the algorithm for "finding the percentage given the percent and base" (which is more familiar and generally taught first) but not the "find the percent" algorithm. Perhaps the test-like environment and an expectation that the problem had to be solved using a computational algorithm prevented students with limited knowledge of percent algorithms from assuming that they could create a mental (or visual) model of the test item, and thus answer the questions without formal computational procedures.

The remaining 10 students (17%) displayed various patterns of responses to questions involving 50%. Included in this group were two students who were able to solve one written computational task (50% x 10=?) but were unable to solve either the equivalent shopping question or the visual task.

Performance on Tasks Involving 25%

We expected relatively high performance levels on tasks involving 25%, not only because 25% appears frequently in daily life, but also because it can be expressed as a familiar fraction ("quarter") and it is numerically related to 50%. However, when students were presented with shopping, visual, and computational tasks similar to those involving 50%, knowledge and performance were much more limited. Table 6 shows percentages of correct responses grouped by grade level.

Table 6

Percentages of Appropriate or Correct Responses (Correct+Ballpark) to Questions About 25%

Question type	Mathematical structure	Grade level: 6th and below n=15	7th-8th n=24	9th and above n=15	Unclassified n=6	Total n=60
Meaning	What does 25% mean?	53%	83%	93%	67%	77%
Shopping	What is 25% off $80?	13% (40%)	33% (50%)	40% (73%)	33% (83%)	30% (57%)
Visual	What is 25% of 20?	47%	63%	80%	83%	65%
Written computation	What is 25% of 20?	27%	58%	53%	33%	47%

Explaining the Meaning of 25%

When asked to explain what 25% means in a department store flyer stating, "25% off sale," 77% of the students provided appropriate responses generally by referring to fractions (one fourth or one quarter), money (25 cents off a dollar), or a combination of fractions and money. ("One quarter" was a difficult response to classify since the students could not always decide if they meant a fractional part, the name of a coin, or both.)

Of the 14 (23%) inappropriate responses, 7 students could not provide any explanation for what 25% means. Seven other students named inappropriate fractions or numbers without being able to explain in mathematical terms why that fraction or number would be equal to 25%, as is illustrated by the following quotes.

Interviewer: How would you explain (25% off) to someone who doesn't know anything about percents?
Charlene: The price is going to be lower, one fifth.
Interviewer: Why is 25% equal to one fifth?
Charlene: From listening to my teacher.

Maria: I would ask my mom. Some would come off but I don't know how much. I guess about $5.
Interviewer: Why $5?
Maria: Just a guess. See the 5 (in 25%), I think $5 off. Could you divide the 25 into the price? Could that be how much money?

Mental Math With 25%

Thirty percent of the students were able to accurately compute mentally the sale price of an $80 coat on sale for "25% off." An additional 27% arrived at responses that were within 10% above or below the correct response (ballpark responses) although they did not necessarily use mathematically meaningful strategies. Four strategies were identified as appropriate because they each would lead, if utilized without computational errors, to a correct solution. Fifty-six percent of the students used one of these four appropriate strategies to solve the "25% off of $80" problem.

- *Benchmark percent strategy* (31% of all students): The student uses "easy to compute" percent (50%) as an intermediate step in the solution process. (*Lisa:* 50% of 80 is $40. Half of that is 20, so I'm breaking my $80 into fourths, that's how I got the 25% off.)
- *Fraction strategy* (20% of all students): The student changes 25% to the fraction one-fourth and then divides $80 by 4.
- *Algorithm strategy* (3% of all students): The student transforms 25% into a decimal, .25, and then mentally multiplies by 80.
- *Amount per unit strategy* (2% of all students): The student identifies 25% as 25 cents out of one dollar. Then the student adds twenty-five cents 80 times. This strategy may be one that is particularly suggested by a monetary situation since a percentage of a dollar can be easily transformed into a certain number of cents, which is a meaningful entity.

Forty-four percent of the students used strategies such as the following, which would not necessarily generate correct solutions to percent problems.

- *Using the percent as a number strategy* (15% of all students): The student ignored the fact that 25% was a percent and used 25 as a number, subtracting it from $80 to get an answer of $55.
- *Partial algorithm strategy* (5% of all students): The student used a combination of portions of a learned computational procedure and other inappropriate procedures. (Depending on the numbers involved, this strategy may result in a reasonable, in-the-ballpark solution.) (*Terry:* It would be about $64. About $16.50 off. I multiplied the 2 with the 80 and got 16. The 5, I just threw an extra zero on it (for the fifty cents).)
- *Proportional size strategy* (5% of all students): The student identified 25% as "pretty small" or "not too much," and then took an estimated proportional amount from the $80. This strategy suggests an appreciation for the proportional nature of percent and with skill can yield a reasonable estimate, but the students did not express a set of constraints to guide their solutions. (*Doris:* I'm estimating, a few dollars off. Twenty-five percent, about 8 or 9 dollars off, give or take a dollar or a few.)
- *Guessing strategy* (18% of all students): The student chose a number "at random" for a solution.

Visual Representation and Written Computation With 25%

Sixty-five percent of the students were able to circle 25% of a pictorial array of 20 stick figures most often by using appropriate, but sometimes inappropriate, strategies. Strategies that led to (or, had they been used without computational errors, could have led to) correct solutions were generally similar to those used in the mental math shopping task described above.

- *Benchmark strategy* (27% of all students),
- *Fraction strategy* (38% of all students),
- *Algorithm strategy* (2% of all students), and
- *Partition 100% strategy* (5% of all students): The student identified the entire array as 100%, determined (often through trial and error) the percent that each of the individual figures represented (5% in this case), then, using the figures as "counters," repetitively added that amount until the target percent was reached.

The inappropriate strategies included:

- *Percent as a number strategy* (5% of all students),
- *Proportional size strategy* (7% of all students), and
- *Guessing strategy* (17% of all students).

The partition strategy would have been unwieldy for the shopping task using $80, but was reasonable for the visual task given the task variables, which included the number 20 (evenly divisible into 100%), discrete objects rather than continuous quantities (money), and the availability of the figures as counters. The incidence of using the percent as a number (transforming 25% into the number "25") was lower for the visual problem with 20 items than for the mental math problem that used $80. The difference in the numbers used in the problem may have discouraged the use of this strategy since it would seem impossible to separate out "25" of the 20 stick figures while it was not impossible to deduct 25 from $80 to find a sale price.

While 39 students were able to answer the "25% of 20 people" visual task correctly, only 19 of them were also able to solve the mathematically equivalent written computational task "What is 25% of 20?". It is possible that the students approached the written assessment with the belief that because the assessment resembled a "school task," only school-like strategies such as algorithmic computation would be appropriate and therefore did not consider using informal or intuitive strategies.

RESPONSE PATTERNS ACROSS TASKS INVOLVING 25%

Of the 60 interviewees, 12 students (20%) responded appropriately to all four different questions involving 25%. Seven people (12%) were unable to respond correctly to any of the questions. The remaining 41 students (68%) exhibited 5 patterns of responses, which are described below.

a. *Success with all tasks except "25% off of $80."* Seven students (12%) were able to appropriately explain the meaning of 25%, solve the visual task, and answer the written computational problem, yet were unable to compute mentally the cost of an $80 coat on sale for 25% off (a real-life task). Some of these students simply subtracted 25 from 80, while some tried to compute 25% of 80 by converting to a decimal notation and then using a multiplication algorithm, but found the procedure awkward without paper and pencil. One student first tried the percentage amount per unit strategy which is suggested by the monetary nature of the problem, then seemed to move towards finding half of a half (benchmark strategy), but was unable to get through the process.

Angela: Taking 25 cents off each dollar and add up what's left so 75 cents eighty times. (pause) I can't.
Interviewer: Can you make a guess?
Angela: Cut that 80 in half, that's $40. Two 25%s is 50 and half off of 80 is 40. So $40.

However, when attempting the visual problem (circle 25% of 20 stick figures), this student was able to complete the process of finding half of a half, probably by relying on the diagram to keep track of the quantities which were too much to keep in working memory while solving the shopping problem.

Angela: Twenty-five minus 20 is 5%, no. (Pause) Here's half, half again. So 5 of them will take it (the GED). The whole picture, split in

half, 10 and 10, split it again.
Interviewer: Why?
Angela: You're not asking for half. Four twenty-five percents make 100.

b. *Success with all tasks except the computational question*. Six students (10% of all students) responded appropriately to all questions except the written computational question ("25% x 20=?"). These students demonstrated knowledge of the meaning of 25% and an ability to deal with 25% in everyday environments, but were less competent when dealing with context-free problems.

c. *Success only with visual task*. Fourteen students (23% of all students) were able to solve the visual problem, but not the mathematically equivalent written assessment problem, nor the shopping problem. These students have some sense of the meaning of 25% as one fourth, but cannot operationalize that knowledge in any way other than by visually dividing a quantity into 4 parts, as illustrated by the two quotes below.

Interviewer: 25% of the (20) people want to take the GED test.
Michelle: Only taking off one quarter of it, by counting by 5. Only got 20, a quarter off of the whole thing. 5, just by looking at it.

Interviewer: Could you figure out how much money this $80 coat costs on sale?
Michelle: $72.50. I divided 25 by 80, leave me with 72 remainder 50.
Interviewer: Why did you decide to do that?
Michelle: Just by looking at it and taking a guess. I averaged it out. I know that 25% is one quarter of that price. One quarter off of $80 is $75. $80 taking one quarter off, only about 5 bucks off of it, so $75.

d. *Success with computational task but not the shopping or visual tasks.* Nine adults (15% of all students) provided appropriate answers to the written assessment question, but not the mental math shopping task nor the mathematically equivalent visual problem. Five of these students explained the meaning of 25% appropriately. Apparently, these students have developed computational skills, but lack a sense of the meaning behind the computations; this hinders them from making sense of situations where the numbers involved are not set out in standard, school-based form.

Interviewer: Could you figure out how much money this $80 coat costs on sale?
Tara: 25 over 100 probably, probably 80 over 100, cross multiply. I could multiply quickly, but I don't have paper. Use 100 amount, take 25%, leave me $75, from that I would need 5 more to make 80. Take another $5 off and come up with 30. Take 25 from 80 and another 5, take 30 from 80, about $50.

e. *Does not operationalize conceptual understanding of 25%.* Five students (8% of all students) provided an appropriate response for the meaning of 25% but did not answer the shopping, visual, or computational tasks involving 25% correctly. Apparently, their intuitions of the meaning of 25% are fragile and thus not really part of an integrated or useful system. The series of responses given by one of these students demonstrates some appropriate ideas that become confused easily.

Interviewer: Here it says "25% off." Can you explain what the 25% means?
Theresa: One fourth off of the price.
Interviewer: Why is 25% equal to one fourth?
Theresa: Four 25s equals 100 and one 25 is one fourth of 100.
Interviewer: Why 100?
Theresa: Just thinking in quarters or 25s. Or $25. 4 of them equal to $100. 25 out of 100 is taking one fourth of the 100.

Shopping problem
Interviewer: Could you figure out how much money this $80 coat costs on sale (with 25% off)?
Theresa: $40, half of, 40 and 40 is 80. Split the 80 in half. Half of 50 is 25, so half of 80 is 40.
Interviewer: Why 50?
Theresa: I broke 80 in half, so I broke the 25 in half. Half of 50 is 25.

Visual problem involving 25%
Interviewer: 25% of the (20) people want to take the GED test.
Theresa: (Circling 10 students), this is one fourth of the students.

Visual problem involving 50%
Interviewer: The 10 people in the circle (out of 20) will work on math today. What percent of the class is that?
Theresa: One half of the class, so 25%. Half of the class is taking math and half of the class is not.
(Theresa did not respond to the written computational 25% question; she solved written questions involving 50% and other percent values by multiplying the percent by the base number, disregarding decimal points and sometimes losing a zero.)

GENERAL TRENDS ACROSS TASKS INVOLVING BASIC PERCENT IDEAS, 50% AND 25%

In all, students were presented with 14 tasks in the interview and written assessment: five tasks concerning knowledge of 100%; five tasks involving 50%; and four tasks involving 25%. Table 7 summarizes performances across these three task categories, and shows the percentage of students who performed successfully (using a criterion of responding appropriately to all or all but one of the tasks within a category (i.e., 4 out of 5 tasks involving 100%, 4 out of 5 tasks involving 50%, or 3 out of 4 tasks involving 25%).

Table 7

Percentages of Students Who Had No More Than One Inappropriate or Incorrect Response for Each Category of Percent Questions

Categories	Total number of questions	Grade level 6th and below n=15	7th-8th n=24	9th and above n=15	Unclassified n=6	Total n=60
Knowledge of 100%	5	60%	63%	80%	83%	68%
50% tasks	5	53%	71%	93%	67%	72%
25% tasks	4	13%	46%	53%	50%	40%

The success rates in the categories of “Knowledge of 100%” and “50% tasks” are quite similar. Of the 60 students, 41 (68%) were successful with “100% questions” and 43 students (72%) were successful with “50% tasks.” When individual performance across these two categories of tasks is considered, 55% of the students were successful in both categories and 15% were successful in neither category. The remaining 30% of the students were successful in one category but not the other, with about half successful in each category.

The finding that 30% of the students were successful in one category but not the other suggests that the ideas targeted by the two categories of questions may not inform each other, meaning that a demonstrated knowledge of one set of ideas or skills does not necessarily lead to knowledge of the other; each body of information or skills is attainable in isolation for these students. Apparently, some students have some knowledge of the different facets of 100%, yet this knowledge does not help them sufficiently to make sense of situations in which 50% appears. Other students realize that 50% is equivalent to one half and are able to apply that knowledge in a useful way, yet do not have an elaborated conceptualization of a system based on 100% within which 50% has meaning. Perhaps the knowledge of the meaning and application of 50% is not mathematically based but was developed through personal experiences and encountering the common usage of percent words in everyday language in which the term “50%” is treated as a word synonymous with “half” rather than as part of a mathematical system.

Table 7 shows that the tasks using 25% were more difficult for students in all groups than were the 50% tasks. Yet, 24 students (40%) were successful with at least three of the four 25% tasks, including 2 students from the group with the lowest scores on the standardized tests. The successful students were found to be those who also demonstrated proficiency on the “Knowledge of 100%” questions (only 1 of the 24 students responded appropriately to *less* than four of the five questions) and on the tasks using 50% (only 2 of the 24 students were not successful here and all of their missed questions were written computations). These data suggest that those who were competent in comprehending and using 25% also demonstrated *both* a knowledge of the role of 100% within the percent system and the ability to use at least one other percent (50%) in a meaningful way.

On the other hand, demonstrated knowledge of the facets of 100% does not necessarily imply an ability to activate that knowledge across a variety of tasks using 25% (for 18 students), nor does an ability to work with 50% necessarily transfer to an ability to work with 25% (for 21 students). Knowledge of 100% and the ability to use 50% appropriately, to the extent these constructs were measured, is apparently not always sufficient for students to be in a position to generalize their knowledge to 25%.

As expected, the highest grade-level group was the most successful with the various percent tasks and displayed a broad knowledge of the role of 100% as the basis of the percent system. However, even within this group, there was evidence of gaps in understanding as well as some limitations on how and when knowledge was applied. Less expected was the ability of many in the lowest grade-level group to respond successfully to many of the questions, particularly those involving 100%. Half of those in the lowest grade group were able to respond appropriately to either 4 or 5 of the questions regarding basic ideas about the role of 100% in the percent system. Many of the adult students who are classified as needing much remedial mathematics education (based on existing testing practices), appear to have some knowledge of the percent system and/or some familiarity with 50%; this knowledge, however, seems to be limited to informal ideas that are not integrated within an elaborated framework of percent knowledge and that therefore do not inform activities involving 25%.

DISCUSSION

When adults reenter an educational setting, they come to instruction with preexisting intuitions and knowledge regarding mathematical concepts and operations, based on their prior formal learning as well as on informal or work-related experiences they may have had. The formal knowledge that adult students bring with them may have been greatly modified or enhanced since it was acquired in school; also, it may not necessarily be part of an elaborated cognitive structure, but rather have been developed within specific cultural or everyday practices and generally remain tied to those contexts (Bishop, 1992; Lave, Murtaugh, & de la Rocha, 1984; Nunes et al., 1993).

Clarification of the nature of adults' preexisting knowledge becomes educationally important if we assume that adults construct new knowledge in part on the basis of their (pre) existing knowledge. It is often argued that the preexisting knowledge students bring with them to the classroom can facilitate, but also hinder, later development of mathematical knowledge and skills (Leinhardt, 1988). However, few detailed descriptions of the mathematical knowledge of adult students in the United States have been published. This study focused on clarifying the nature of adults' informal and formal knowledge about percent, a concept having wide application in home, work, and citizenship life contexts of adults. Research on adults' preexisting knowledge about percent would help illuminate issues in assessment and accommodation during classroom instruction on percent. By looking at adults' patterns of responses to percent questions, we establish the breadth, depth, and range of their preexisting knowledge and describe the environment into which new instruction must be integrated.

RESPONSE PATTERNS

Virtually all of the adults who participated in this study, including those with low prior educational achievement or "grade level" as determined by standardized test scores, demonstrated some conceptual understanding of percent upon entering their programs. However, response patterns to explanatory, shopping, visual, and computational tasks varied considerably, both within and across students. Only six students (10%) responded appropriately to all tasks, even though these tasks involved percent values (100%, 50%, 25%) that many educators would consider simple and that we might expect adults to understand given that they have to deal with such percent values almost daily.

Students classified at the higher grade levels showed stronger percent-related knowledge and skills than students classified at lower grade levels. Such a performance difference is not surprising and could be expected, in part because performance on both program-administered tests and the written assessments used in this study are determined to some extent by the same factors, such as knowledge of school-taught computational algorithms or test-wiseness.

While students at the higher grade levels performed better as a group, many of them also showed gaps in their knowledge of percent, whether in knowledge of 100%, in informal mental math, or in computation. At the same time, reasonable performance on some tasks was also found among a sizable, though smaller, group of students classified in the lowest grade level (6th and below). Between one third and one half of all students (depending on which percent task was used) performed better on mental and visual tasks than on mathematically equivalent written computational tasks. A smaller group did better on computational tasks (probably by using school-based algorithms) than on mental or visual tasks that required more conceptual understanding and flexible problem-solving strategies.

KNOWLEDGE ABOUT THE PERCENT SYSTEM

The basic principle underlying the percent system, that proportions can be expressed as quantities in relation to a standard number (100), appeared familiar in at least one context to nearly all participating students (who have not yet studied about percent in their programs). However, students' knowledge of 100% was not robust; several students who, in one context, appeared to understand the principle that percents are interpreted in relation to a 100-unit system nevertheless became hesitant or unsure about it in other contexts. Lembke and Reys (1994) reported a related finding: eleventh-grade students showed less confidence in 100 as the basis for percent than did younger students, even though the older students had the benefit of more formal instruction in percents.

For many of the students in this study, "100%" appeared as a convenient, commonly used, round number which, in at least some contexts, did not specifically imply a whole that is always divisible into 100 parts. Some students' interpretations of "100% apple juice," for example, were along verbal rather than mathematical lines and were incompatible with statements that they made about the meaning of 100% in other contexts. Similarly, while most students had little or no difficulty interpreting the term "50%" as "half" and many performed mental computations with 50% without errors, some were unable to give a mathematical justification for why 50% is equal to half.

We expected that most students who know that 50% is a "half" would also realize that 25% is "half of a half," but this was not the case. Performance on tasks involving 25% was markedly lower than on tasks involving 50%. Many students identified 25% as "one fourth," or invoked explanations couched in their experience with the American monetary system (e.g., "25% is equal to a quarter because there are four quarters in a dollar"); they often did not connect this knowledge of facts about fractions or money to the knowledge of the percent system that they may have demonstrated on other tasks.

This pattern of results gives rise to the hypothesis that many adults who come to literacy programs may have developed certain verbal associations involving percents through repeated encounters with percent terms within everyday discourse (e.g., "I feel 100%," "100% natural"). As a result, they have not fully grasped the mathematical principle underlying the meaning of such terms in each specific context and across different contexts.

The teaching and learning challenges caused by the existence of both everyday and mathematical meanings for terms used in the mathematics classroom have been the focus of previous work (e.g., Laborde, 1990). The case usually made is that students (or teachers) apply a legitimate everyday meaning of a term instead of a mathematical meaning, or vice versa, causing confusion in classroom communication and disrupting understanding. This study showed a related but different phenomenon: students associate 100% with "all," "50%" with "half," or "25%" with "quarter," but may not fully understand the mathematical relationship between 100% and 50%, or between 50% and 25%. Each term may gain an idiosyncratic meaning for some students, based on the context in which it was encountered. Under such circumstances, uneven performance on different tasks, of the kind observed in this study, is quite possible.

Implications for Instructional Practice

Adult mathematics instruction in percents aims to help students develop the knowledge and skills that will be useful in a variety of contexts, both within and outside of the learning environment. Since adult students already function in the real world, new learning should enhance or improve their existing numeracy practices as well as enrich their understanding of the mathematical foundation that supports these practices. Effective instruction requires attention to and analysis of existing knowledge and practices.

Content of Instruction

As Resnick (1986) and others (e.g., DeVault, 1991) have pointed out, informal knowledge, regardless of its degree of correctness and usefulness, is self-evident, obvious, and easily accessible to the individual holding it, and is embedded in the everyday practices in which it develops. Nunes et al. (1993) observed that knowledge developed in out-of-school contexts is less accessible to reflection (i.e., its holder does not have reason to understand the mathematical generalities that underlie it). Therefore, instruction of adult students should encourage them to reexamine the meanings that they attach to familiar terms, helping them to see the mathematics within and underlying these terms while promoting the development of habits of reflecting on the mathematical implications of their everyday practices.

Gal (1993) has noted that professional development resources for adult educators often do not acknowledge the potential influence of learners' informal mathematical knowledge on the teaching-learning process. Textbooks and workbooks aimed at the adult education market usually focus on developing students' computational skills, and offer few opportunities for students to connect classroom exercises with their out-of-school knowledge and practices. Many textbooks present the study of percent as an extension of the study of fractions and decimals. This approach leads to an emphasis on the mechanics of percent-related calculations, or on converting between percents, fractions, and decimals, rather than also on (a) developing an intuitive "feel" for the ordinal properties or meaning of percent (e.g., that 90% is "pretty close" to 100%), (b) developing a robust understanding of the meaning of percent in multiple contexts, and (c) developing interpretive skills appropriate for percent-laden everyday contexts where computation is not required.

Interpretive skills are seldom emphasized in textbooks or in professional development resources for teachers (Gal, 1993), even though adults' encounters with percents (e.g., in newspapers or financial documents) so often require comprehension and sense-making rather than computation. Our current study shows that many students can make appropriate interpretations of some percent statements, even when their formal or mental computation skills are lacking. Continued development of interpretive skills (including students' mathematical vocabulary and oral skills) may require better integration of literacy and numeracy instruction, including the use of everyday reading materials that mention percent (e.g., newspapers), rather than expecting interpretive skills to develop informally as a by-product of attention to decontextualized computational skills.

USING ASSESSMENTS TO INFORM INSTRUCTION

Standardized tests such as the TABE or ABLE were used by all the programs that participated in this study; they are being used by most adult education programs in the United States (Gal & Schuh, 1994) to evaluate students' mathematical skills. The discussion above suggests that sole reliance on written assessments can provide incomplete or distorted estimates of students' pre-instruction knowledge of percents (and other mathematical skills), and misinform decisions about starting points for instruction (Streefland & van den Heuvel-Panhuizen, 1992).

Students' true knowledge of percent may at times be masked by ambiguities caused by their ability to employ convincingly percent terms based on their colloquial, non-mathematical use of such terms outside the classroom. This may cause teachers to overestimate students' knowledge, leading to erroneous instructional decisions. Assessment of percent-related (and other numeracy) skills either at the onset or during instruction, should therefore be enhanced by including open-ended, realistic performance tasks or simple simulations, which would aim to elicit students' informal as well as formal knowledge. Care must be taken to evaluate not only responses to questions involving familiar benchmark percents, such as 50% or 25%, but also involving less familiar percents (e.g., 15%, 7%, 1%, 150%). These percents are less likely to trigger everyday verbal associations that may displace mathematical meaning for the student or suggest an inaccurate assessment to the instructor.

The difficulties many students displayed in explaining their thinking about percent raise doubts about the appropriateness of using self-assessments as a diagnostic tool; such assessments are currently used by some adult literacy programs in the United States during an intake process (Gal & Schuh, 1994). Students may be unaware of the patchy or superficial nature of their own knowledge of percents and, being (mis)led by their familiarity with context-specific applications of percent, may overestimate their knowledge and skills.

CONCLUSIONS

An increasing number of mathematics educators argue that instruction will be most effective if teachers acknowledge and build upon intuitions and informal understanding that students bring to the classroom (Lacampagne, 1993). This study provided examples of strengths and weaknesses in adult students' pre-instruction knowledge of percents; adult educators should be cognizant of this situation when planning instruction or when interacting with students. Our findings further suggest that the learning of mathematics too often occurs in isolation from its applications, and many functional uses of numeracy skills go unrecognized and unsupported in the mathematics classroom. Both practitioners and researchers should continue to explore effective methods for ensuring that students' classroom-based skills inform their everyday quantitative practices and interpretive skills, and vice versa.

The knowledge that teachers may glean about students' informal and formal skills regarding percent (and other mathematical subjects) is of limited value unless teachers make use of it in their instruction. Adult educators are pressured by the need to prepare many students for passing tests (e.g., GED) and by the need to work with students within the same class who have a variety of abilities and backgrounds. Under such circumstances, many educators may find it difficult to assess and accommodate students' preexisting knowledge. (A similar situation may exist in K-12 education. Leinhardt (1988), for example, describes a case of master mathematics teachers who ignore elementary school students' informal knowledge.)

Improving methods for assessing the full range of students' preexisting mathematical knowledge is only a first step on the way to improving practice. This study showed that complex and diverse profiles of student knowledge of percent do exist. Professional development efforts should therefore encourage educators to consider the specific ways in which they can adapt their instruction, depending on the results of improved assessments.

Modification of the teaching-learning process by taking into account adult students' prior knowledge will face many challenges. The implementation of a new teaching perspective has been shown to involve a lengthy but worthwhile experimentation process by teachers (Leonelli, Merson, & Schmitt, 1994). Educational organizations must be willing to support teachers as they explore new instructional and assessment practices. Just as our understanding of adults' mathematical knowledge and learning is evolving, educational practice should also continually evolve to ensure the most effective and meaningful educational experiences for adult students.

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