MAA
MATHEMATICAL ASSOCIATION OF AMERICA

Quantitative Reasoning for College Graduates: A Complement to the Standards

This Report, originally published in 1994, reflects some of the earliest efforts to reshape the notion of quantitative literacy for a rapidly-growing, and substantially changing, student body. While much work has been done since, we have maintained the report to provide insight into the development of more-recent quantitative literacy/numeracy efforts.

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Summary

What quantitative literacy requirements should be established for all students who receive a bachelor's degree? Over the years, the Mathematical Association of America (MAA) has approached this question in various ways, most recently by establishing, in 1989, a Subcommittee on Quantitative Literacy Requirements (henceforth called the *Subcommittee*) of its Committee on the Undergraduate Program in Mathematics. The work of the Subcommittee has been similar in some respects to the efforts of the National Council of Teachers of Mathematics (NCTM) that led to its celebrated *Curriculum and Evaluation Standards for School Mathematics* (1989) and related publications. The recommendations from the Subcommittee can be

considered to complement those in the *Standards*. They also should be viewed as a reasonable extension of a Standards-based high school experience to the undergraduate level.

The Subcommittee began with the perception, supported by many recent studies and reports, that general mathematical knowledge among the American people is in a sorry state. It assumed that colleges and universities would welcome some suggestions on what they might do about the situation.

The discussions and investigations conducted by the Subcommittee led to four primary conclusions. The conclusions embody a vision that goes well beyond present practice in most places.

Conclusion 1. Colleges and universities should treat quantitative literacy as a thoroughly legitimate and even necessary goal for baccalaureate graduates.

Many authoritative mathematical and other groups have affirmed the importance of quantitative, or mathematical, skills in the population at large. These skills are valuable in various ways (this report lists nine), e.g. in daily life, further education, careers, and overall citizenship. To some degree these skills are acquired by the end of secondary education, but the post-secondary experience should reinforce what has been learned in school and go beyond. Thus the Subcommittee's concern has been not with quantitative literacy in general, but with *quantitative literacy for college graduates*, which naturally should differ in both depth and quality from that expected of high school graduates.

Conclusion 2. Colleges and universities should expect every college graduate to be able to apply simple mathematical methods to the solution of real-world problems.

Rote and passive learning of mathematical facts and procedures is not enough. Educated adults should be able to interpret mathematical models, represent mathematical information in several ways, and use different mathematical and statistical methods to solve problems, while recognizing that these methods have limits. These elements extend those in the ideal of "mathematical power" presented in the NCTM *Standards*, which include "methods of investigating and reasoning, means of communication, and notions of context." At the same time, these goals seem attainable.

Conclusion 3. Colleges and universities should devise and establish quantitative literacy programs each consisting of foundation experience and a continuation experience, and mathematics departments should provide leadership in the development of such programs.

A required course or two is not sufficient. A student becomes quantitatively literate through a broad program that instills certain "long-term patterns of interaction and engagement." The *program*, the central idea of these recommendations, starts with a "foundation experience" into which students are appropriately placed and in which a carefully chosen course or two can raise entering students to a level of proficiency where they can benefit from the next phase, which is the "continuation experience."

In the continuation phase, later in their undergraduate programs students exercise and expand the elements of quantitative literacy they have already learned in the foundation experience and elsewhere. This phase is made possible by a framework of *mathematics across the curriculum*, an array of courses (both within and outside mathematics) and other educational experiences designed, in content and style, to contribute to the strengthening of quantitative literacy. The mathematics should be taught in context. Instructional materials should be current, practical, and conducive to active student involvement. Writing, student collaboration, and thoughtful use of instructional technology all have potentially important places. The program may also include the provision of mathematics clinics and other such resources.

In the course of these efforts, the needs, backgrounds, and expectations of people who in the past have tended to have special problems with mathematics should not be overlooked. Indeed, a well-designed quantitative literacy program may be of exceptional benefit to those persons who have special difficulties with mathematics.

Conclusion 4. Colleges and Universities should accept responsibility for overseeing their quantitative literacy programs through regular assessments.

A quantitative literacy program should be managed watchfully. At appropriate times and in appropriate ways, the results should be evaluated so as to obtain enlightened, realistic guidance for improvement. Evaluation methods should reflect course goals and teaching methods used, and besides pointing to possible improvements in the program can themselves be educationally beneficial. In particular, the evaluation methods should involve clearly applications-oriented tasks.

[The report concludes with five appendices including references, a list of topics on which one might base a reasonable syllabus, brief descriptions of some existing foundations courses, a questionnaire for assessing attitudes toward mathematics, a list of problems related to minimal competency, a set of project ideas, several scoring guides, and comments on approaches to quantitative literacy for two specific majors.]

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Preface

From its inception the Mathematical Association of America (MAA) has sought to improve education in collegiate mathematics. For the past forty years, the natural MAA vehicle for interest in mathematics for general education has been its Committee on the Undergraduate Program in Mathematics (CUPM). Indeed, one of the first fruits of CUPM (then still called CUP) was some material produced under its sponsorship in 1954-1958 under the title *Universal Mathematics*, in two parts. According to the preface to Part I, "Universal Mathematics" has been designed as a course for all first-year college and university students with normal high school preparation in mathematics. Normal preparation includes at least two, and preferably two and one-half, units of high school mathematics ..." (p. iii). The goal, presumably, was a kind of quantitative literacy, but that term was not yet used.

There was some pilot-testing of *Universal Mathematics*, but, especially for some years after receiving its first NSF grant in 1960, CUPM concentrated almost all of its efforts on problems related to more narrowly defined clienteles, and indeed treated "quantitative literacy" in a somewhat gingerly manner. For example, the CUPM booklet *A General Curriculum in Mathematics for Colleges* (1965), which was in some ways a synthesis of the CUPM recommendations that had by then appeared, included an interesting but inconclusive discussion (pp. 25-26) of the issue, punctuated with disclaimers like "These remarks do not have the force of a recommendation, since CUPM has not yet considered in detail this important curricular problem" (p. 25).

It was not until January, 1978, the CUPM formed a panel (subcommittee) to consider the quantitative literacy problem straight on, and this panel published a brief, worthwhile, thoughtful, but also somewhat inconclusive report, "Minimal Mathematical Competencies for College Graduates" in *The American Mathematical Monthly* in April, 1982. A reprint of that report appears in the 1989 MAA Notes volume, #13.

In the last ten years mathematics education in American schools and in colleges has received widespread attention. In response to claims of weaknesses in mathematics education nationwide, a coordinated effort is being made by the mathematical community to set standards for curriculum and teaching, as well as to develop better procedures for determining the extent to which the established standards are being met. In this context the National Council of Teachers of Mathematics has produced two major and influential reports regarding mathematics education for students from kindergarten through grade twelve: *Curriculum and Evaluation Standards for School Mathematics* (1989) and *Professional Standards for Teaching Mathematics* (1991).

The present report is concerned with quantitative literacy requirements that should be established for all students who receive a bachelor's degree. It has been prepared by a CUPM Subcommittee on Quantitative Literacy Requirements which was formed in late 1989 and has worked since then to frame recommendations which both mesh with the new pre-college standards and are realistically achievable in the college years. Implementation of this report's recommendations and ideas can help develop graduates who are quantitatively more literate, with important benefits to themselves and to the society in which they live.

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Introduction and Report Overview

It is no secret that too many educated people exhibit (even flaunt) great deficiencies in basic mathematical knowledge and skills -- they are quantitatively illiterate! How concerned are our colleges and universities about this form of illiteracy? In the last ten years national report after national report has confirmed this fact. What can, or indeed will, the mathematical community do about it? The Mathematical Association of American (MAA), through CUPM, decided to take up the challenge and offer recommendations which could lead to the acquisition of quantitative literacy by a larger proportion of the nation's college graduates. Hence the present report, which is addressed to provosts and deans at colleges and universities, to members of the mathematical community, and to those who serve on general education committees within colleges and universities or are in other ways concerned about the quantitative literacy of college graduates.

The CUPM Subcommittee on Quantitative Literacy Requirements has wrestled with many complex problems that surround quantitative literacy for all college graduates. This report is the product of many lengthy discussions, a focus group conference on the subject sponsored by the National Science Foundation, presentations on quantitative literacy and reactions to them at national and state professional meetings, in-depth study of national reports and data, and much input by individual mathematicians, scientists, teachers, college administrators, and staff members of state boards of higher education. It attempts to provide workable solutions to many complex problems and guidance toward attainment of those solutions.

The report is visionary in that it does NOT represent a distillation of current national practice in supplying college students with mathematical training. Rather it sets a standard for a quantitatively literate college graduate and suggests reasonable means for the achievement of that standard.

The Subcommittee looked at the problem of too few college graduates being quantitatively literate by asking: why should college graduates be quantitatively literate? what should a quantitatively literate college graduate be able to do? what mathematical topics and experiences would support development of these capabilities? how can colleges and universities realistically proceed from their current curricular programming to provide ways for their college graduates to become quantitatively literate? and what procedures can be used by colleges and universities to assess the extent to which they are accomplishing their goals? The Subcommittee has summarized its conclusions in the present report, which presents a challenge to the will and dedication of those in the mathematical community as well as other faculty and administrators at colleges and universities across the nation.

The report makes four major points which are set forth in detail. They are:

1. Colleges and universities should treat quantitative literacy as a thoroughly legitimate and even necessary goal for baccalaureate graduates (see Part I);
2. Colleges and universities should expect every college graduate to be able to apply simple mathematical methods to the solution of real world problems (as described more fully in Part II);
3. Colleges and universities should devise and establish quantitative literacy programs each consisting of a foundation experience and continuation experiences (see Part III), and mathematics departments should provide leadership in the development of such programs;
4. Colleges and universities should accept responsibility for overseeing their quantitative literacy programs through regular assessments (see Part IV).

The cardinal recommendation among these four is the establishment of a quantitative literacy program -- not a course! Many colleges and universities have mathematics requirements in their general education programs. Often this requirement is a choice of one course from a list of possible entry level courses at that institution. No rationale is given for the course and nothing is said about its relationships with other courses or with the remainder of the student's program at the college or university. Students are naturally led to the idea that the course is merely a hurdle to jump, and then its content might as well be forgotten.

Basic quantitative literacy depends on students being introduced to the foundations of quantitative reasoning and then given reinforcement experiences which develop and deepen in the student the habits of thinking which the student has been encouraged to develop. Taking one course is not enough to endow a student with a habit of mind, but completing a carefully devised program can provide sufficient practice to make a pattern of thought part of the student's intellectual tools. The construction of such a program requires leadership from the mathematics faculty and other faculty as well as commitment to the three other major points of this report.

Many factors need to be considered in determining the degree of quantitative literacy appropriate for all college graduates. Among them are the increasing complexity of the society, the desire to improve opportunities for all citizens to participate more fully in their society, current efforts to improve the standards of school mathematics, and the pressures on colleges and universities for accountability regarding undergraduate education. All of these factors influenced the creation of this report.

Part I summarizes published opinion on the importance of quantitative literacy, discusses some of the reasons for it, and emphasizes that this report's concern is with quantitative literacy specifically for college graduates. Part II explores the elements of quantitative literacy. Part III elaborates on facets of Part II and is concerned especially with the practical problem of developing programs of teaching for quantitative literacy. Part IV deals with the important issue of assessing the effectiveness of quantitative literacy programs. The report concludes with a brief bibliography and some other supporting material.

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Part I: Why Quantitative Literacy?

There seems to be wide agreement that a well educated citizen should have some significant proficiency in mathematical thinking and in the most useful elementary techniques that go with it. In western civilization, the idea goes back at least to classical times, when four (the "quadrivium") of the seven liberal arts considered essential for the education of a free citizen were essentially mathematical. The role of mathematics was enlarged by the Enlightenment, by the Industrial Revolution, and by many events in modern science, technology, business, and the rapid intellectual evolution of humanity generally.

In recent years, amidst intense scrutiny and sometimes harsh criticism of the whole educational system in the United States, one group after another has expressed itself on the point.

A representative statement (here considerably abbreviated) appears in the influential report *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (1989):

> *To function in today's society, mathematical literacy- -what the British call "numeracy"--is as essential as verbal literacy ... Numeracy requires more than just familiarity with numbers. To cope confidently with the demands of today's society, one must be able to grasp the implications of many mathematical concepts--for example, change, logic, and graphs--that permeate daily news and routine decisions--mathematical, scientific, and cultural--provide a common fabric of communication indispensable for modern civilized society. Mathematical literacy is especially crucial because mathematics is the language of science*

> *and technology ..."*

An emphasis on the expanding importance of general education in mathematics beyond high school was made over twenty years earlier, in the "COSRIMS" report *The Mathematics Sciences: A Report* (1968), p. 56:

> *"The impact of science and technology has become so significant in our daily life that the well- educated citizen requires a background in the liberal sciences as well as the liberal arts. It has long been recognized that mathematical literacy is an important goal of all liberal education. But in current education this training often stops at the secondary- school level. With the increasing quantification of many of the newer sciences, the impact of high-speed computers, and the general expansion of the language of mathematics, it becomes increasingly important for the college graduate to have some postsecondary training in mathematics ..."*

Or consider the following words from *The Mathematics Report Card: Are We Measuring Up?* (1988) p. 9:

> *"Looking toward the year 2000, the fastest-growing occupations require employees to have much higher math, language, and reasoning capabilities than do current occupations. Too many students leave high school without the mathematical understanding that will allow them to participate fully as workers and citizens in contemporary society."*

Those who have been pleading for more nearly universal quantitative or mathematical literacy have not all been mathematicians, by any means. Consider the words from *50 Hours: A Core Curriculum for College Students* (1989) p. 35:

> *"To participate rationally in a world where discussions about everything from finance to the environment, from personal health to politics, are increasingly informed by mathematics, one must understand mathematical methods and concepts, their assumptions and implications."*

These statements and many others like them add up to an interesting challenge, and since about half of American colleges and universities have no general mathematics requirement for graduation, the challenge is clearly not being met.

There have been encouraging signs of improvement in recent years, but optimism can be premature. As these words are being written, it was just announced by The College Board that the average quantitative score on the SAT has taken another downward turn, after more than a decade without any decrease.

We have been speaking of *mathematical* attainments. The term "quantitative literacy" has so far appeared only in the title. Whether there is a real difference between "quantitative literacy" and "some significant proficiency in mathematical thinking and in the most useful elementary techniques that go with it" is a matter of debate. Sometimes the term "quantitative literacy" is a virtual euphemism for some level, usually ill defined, of accomplishment in mathematics. (How unfortunate that some people should consider it expedient to use a euphemism for "mathematics"!) At other times "quantitative literacy" is used much more broadly, to include logic, linguistics, and other subjects that have at least a relatively formal character, even if they are seldom or ever taught in mathematics departments.

Here we shall adopt the point of view that "quantitative literacy" primarily concerns mathematics, broadly understood. It is not an entirely fortunate term. For one thing, much of modern mathematics, even at elementary levels, is not distinctively quantitative; for another, "literacy" suggests both facility with {\it letters} and a possibly very low level of accomplishment. The term "numeracy" is shorter, at least.

Most, if not all, of what will be said here will apply whichever reasonable interpretation of the term "quantitative literacy" is adopted.

It may be useful to enumerate some of the principal reasons for expecting quantitative literacy of educated people. The list that follows is surely not complete, and the items in it are not independent; but it directs attention to some of the major areas in the broad range of "Why study mathematics?"

1. Mathematical thinking and skills are of great value in everyday life. "Other things being equal, a person who has studied mathematics should be able to live more intelligently than one who has not. And, up to a point at least, the more mathematics studied, the more intelligent the life should be" (NCTM, *A Source book of Applications of School Mathematics* (1980), Preface).
2. One of the classic reasons for studying mathematics is that it strengthens general reasoning powers, for instance by developing problem- solving skills. While the research literature is ambiguous on this point, many thoughtful people are convinced that it is true in some sense.
3. Quantitative literacy at varying levels is clearly needed in preparation for further study in many academic and professional fields. It is reliably estimated that the majority of undergraduates would be required to take a course or courses in the mathematical sciences for this purpose even in the absence of a general graduation requirement of this kind.
4. Increasing amounts of mathematics are needed in an increasing number of careers. "... more and more jobs--especially those involving the use of computers--require the capability to employ sophisticated quantitative skills. Although a working knowledge of arithmetic may have sufficed for jobs of the past, it is clearly not enough for today, for the next decade, or for the next century" (*Moving Beyond Myths: Revitalizing Undergraduate Mathematics* (1991) p. 11. And students, even college seniors, often do not know what careers they will enter, or where their career paths will lead them. A quantitative literacy requirement helps to hold some doors open.
5. Many adults, and especially college graduates, are very likely to assume positions in their communities and in professional organizations where quantitative literacy (e.g., the ability to deal intelligently with statistics) will come into play and may even be essential for effectiveness. A quantitative literacy requirement can thus be expected to enhance the quality of citizens.
6. Anyone who does not have a mature appreciation of mathematics misses out on one of the finest and most important accomplishments of the human race. A quantitative literacy requirement, sensibly defined, will contribute to the spread of that appreciation.
7. Society can ill afford to underdevelop latent mathematical talent. For many students the activities leading to satisfaction of a quantitative literacy requirement can be revelatory, inspiring them to consider for themselves careers in mathematics or mathematics-related fields.
8. The fear of mathematics that is often called "math phobia" or "math anxiety" stunts the cognitive development of those who suffer from it. It is usually learned, not inborn, and a curricular component devoted to promoting quantitative literacy, if competently and compassionately taught, can be powerfully therapeutic against it.
9. In particular, negative attitudes of parents and teachers (including guidance counselors) toward mathematics are all too easily picked up by the next generation. Statements like "Oh, I never was good at math myself" or "Just get this math out of the way and then forget it; you'll never need it again" or "For punishment, you will have to do thirty extra math problems" can do enormous amounts of mischief.

Even if, as many thoughtful people believe, the educational process that finally produces college graduates should be regarded as seamless, practical considerations require that some line should be drawn between the pre-college part and the college part, or in other words between the secondary part and the tertiary part. The present study is sponsored by the Mathematical Association of America, which by its charter is concerned with "collegiate mathematics," so is concerned mainly with the college part.

The term "remedial" (or "developmental"), as applied to a college mathematics course, has a definite meaning only where there is a clear understanding of where precollege mathematics leaves off and collegiate mathematics begins. There are various opinions about where this line may be. However "remedial" is defined, the volume of remedial instruction to college students has certainly increased in the past several decades. According to *A Challenge of Numbers: People in the Mathematical Sciences* by Bernard L. Madison and Therese A. Hart (1990) p. 29.

> “ *In fall 1970, college enrollments in remedial courses constituted 33% of the mathematical sciences enrollments in two-year colleges and by 1985 had increased to 47%. In four-year colleges and universities, remedial enrollments constituted 9% of the mathematical sciences enrollments in 1970 and had*

> *increased to 15% by 1985.*"

In spite of the volume of resources being poured into the teaching of such courses, there is widespread skepticism, backed up by some empirical studies, about their effectiveness, especially in preparing students for genuinely college-level mathematics courses. One should expect more from a quantitative literacy program for undergraduates.

But *is* there an intrinsically "college" part for all students? If agreement can be reached on what "mathematical methods and concepts, their assumptions and implications" every college graduate should understand, does it really matter whether that understanding is acquired before or after matriculation in a college or university? Is it not imaginable that, for example, the goals set for secondary mathematics in the NCTM *Curriculum and Evaluation Standards in School Mathematics* (1989) define an acceptable concept of quantitative literacy ? And if so, and if the *Standards* are widely adopted, will there be anything left for the colleges and universities to do in this area beyond supplying suitable remedial experiences for those students who slip through the cracks? To put the matter another way, is it not imaginable that any quantitative literacy appropriately required for a bachelor's degree should in fact be regarded as an appropriate requirement for admission to a college or university?

There are several very large "ifs" in the preceding paragraph. They relate to difficult questions of definition, curricular diversity and inertia, a great lack of homogeneity in the student population, and other inconveniences. A more important consideration, perhaps, relates to the nature of the postsecondary experience. College students, on the average, are more mature, more experienced, and more thoughtful about their personal goals than they were before they became college students. One does not need to invoke William Perry's scheme to justify a belief that college students should be better able to acquire, and to acquire more deeply, quantitative literacy in any reasonable sense. Indeed, because of the pervasiveness of mathematical ideas in the careers that college graduates usually enter, they should be *expected* to have acquired them more thoroughly and meaningfully than if they had not gone to college.

These ruminations are leading relentlessly to the conclusion that it might be a mistake to speak of "quantitative literacy" as if it were a single, monolithic idea. Surely there are meaningful *degrees* of quantitative literacy, and perhaps it would be useful to identify some of them. Here, we speak of only one--the degree of quantitative literacy appropriately expected of all *college* graduates. As we have suggested, we do not believe that this is identical with the degree of quantitative literacy appropriately expected of all *high school* graduates, even as implied in such a forwardlooking statement as the NCTM *Standards*.

Thus the present report is based on the assumptions that, for many reasons, some significant level of quantitative literacy is desirable in all adults; that the amount appropriate for college graduates is greater than that to be expected at the time of graduation from high school; and that the difference is not merely a matter of "remediation."

Cultivation of quantitative literacy at any level is, of course, a matter of teaching and learning. And teaching and learning involve far more than mere identification and communication of appropriate content. There is ample evidence that the traditional "lecture and listen" mode of instruction, still probably far more the rule than the exception in American higher education, does not work as well as some other modes--certainly not as well as it should. Particularly for those students who are studying in the mathematical sciences not by their own choice, teaching and learning styles that include active involvement, cooperation, and the personal touch are much to be preferred over those that do not.

So while the emphasis in this report will be on what the elements of quantitative literacy are, we also implore those who are responsible for providing students with classes and other opportunities for developing quantitative literacy to give a great deal of attention to the form those opportunities should take and the manner in which they should be delivered.

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Part II: Quantitative Literacy: Goals

The foremost objective of both liberal and professional types of higher education should be to produce well-educated, enlightened citizens, who can reason cogently, communicate clearly, solve problems, and lead satisfying, productive lives.

What should every college graduate know about mathematics? ``What would you teach [undergraduate] students if they took only one term [or two terms] of math during their entire college career[s]?'' (*For All Practical Purposes*, (1988)).

No college course can make up for years of neglect or misdirection in earlier mathematics education. If the NCTM {\it Curriculum and Evaluation Standards for School Mathematics} are widely adopted and pursued relentlessly, eventually there will be no need to try. Until they are, however, remediation will be a necessary component of quantitative literacy efforts at most colleges and universities. At this level mathematics that has been encountered unsuccessfully over and over again must be presented ``subversively.'' Make it look fresh and different! Significant mathematical ideas and techniques should be developed within compelling applied contexts as natural, powerful tools for understanding and description. If no genuine application of a topic can be found at the appropriate level, omit it.

It has been suggested that every college mathematics course should be conceived as though it were to be the students' last--for in most cases, it will be (``Reaching for Science Literacy,'' Lynn Steen, *Change*, July/August, '91). Understandably, the prevailing view of mathematics condemns it as nothing more than a confusing set of rules for manipulating symbols out of context and naming geometric shapes. It is seen as old, static, and intellectually confining. Any attempt to achieve quantitative literacy must refute these stereotypes by portraying mathematics as broadly useful in contemporary life connected to learners' experiences and by stressing the active, experimental, open-ended aspects of mathematical thinking and mathematical problems. It is becoming clearer and clearer that how people learn is just as important as what they learn. In order to build confidence in our students that they can read and learn mathematics on their own, it is essential that we set good examples by the pedagogical strategies we choose. Replace lectures as the primary means of delivery with more active, engaging experiences. Require teamwork, discussion and writing about mathematics. Emphasize making sense of inherently quantitative situations, problem formulation and heuristics at the expense of mechanics. Insist that calculators and computers be used routinely to carry the burden of computation and for intelligent exploration.

Any effective attack on the problem of quantitative literacy must recognize that not all mathematical roads are narrow, algebraic ones that lead to calculus. Today's routes must offer glimpses of a broad mathematical landscape with applications prominent in the foreground. To achieve some depth along the way, college students must be taught to view landmarks from a variety of perspectives-- numerical, visual, verbal and symbolic. They must learn that understanding, explanation and prediction are the real mathematical destinations, not the answers in the backs of textbooks. Unless we repeatedly immerse students in interesting quantitative settings that require drawing inferences from data, interpreting models, estimating results, assessing risks, suggesting alternatives, and even making reasonable, testable guesses, students will never see the forest for the trees.

In short, every college graduate should be able to apply simple mathematical methods to the solution of real-world problems. A quantitatively literate college graduate should be able to:

1. Interpret mathematical models such as formulas, graphs, tables, and schematics, and draw inferences from them.
2. Represent mathematical information symbolically, visually, numerically, and verbally.
3. Use arithmetical, algebraic, geometric and statistical methods to solve problems.

4. Estimate and check answers to mathematical problems in order to determine reasonableness, identify alternatives, and select optimal results.
5. Recognize that mathematical and statistical methods have limits.

These five capabilities could be attained at varying levels. In particular the level intended here is beyond that normally attained in the high school experience. Explicitly college-bound high school students are generally expected to have three, and encouraged to have four, years of college preparatory high school mathematics. A quantitatively literate {\it college} graduate should be expected to have deeper and broader experiences than those who only graduate from high school. The level of sophistication and maturity of thinking expected of a college student should extend to a capability for quantitative reasoning which is commensurate with the college experience. College students should be expected to go beyond routine problem solving to handle problem situations of greater complexity and diversity, and to connect ideas and procedures more readily with other topics both within and outside mathematics.

Some guidance here can be attained from the manner in which the NCTM *Curriculum and Evaluation Standards* treat their school goals of problem solving, communication, and reasoning. These three standards persist throughout their K-12 curriculum but ``details vary between levels with respect to what is expected both of students and of instruction. This variation reflects the developmental level of the students, their mathematical background, and the specific mathematics content'' (p. 11). In one sense quantitative literacy for *college* students may be seen as *extending* the notion of mathematical power described in the NCTM *Standards* -- ``it includes methods of investigating and reasoning, means of communication, and notions of context'' (p. 5) -- to the intellectual developmental level expected in a post-secondary education. Although quantitative literacy for college students would include some mathematical content, it especially involves the ability to use concepts, procedures, and intellectual processes. It should also include a degree of versatility in approaching and solving problems.

Too big an order for a one- or two-term mathematics course? Unquestionably. Just as writing is not the sole province of English departments, neither does the responsibility for students' mathematical development rest only with mathematicians. Of course, the impetus to promote quantitative literacy , the leadership to define its elements effectively, and the energy to sustain its objectives will have to reside in the mathematical community. But mathematics must permeate the undergraduate experience the same way it permeates modern society: MATHEMATICS ACROSS THE CURRICULUM!

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PART III: Actions and Strategies

Concerned with aspects of developing a quantitative literacy program, this part consists of somewhat disjoint sections. The theoretical framework for the development of problem solving is discussed and the concept of mathematics across the curriculum is described. The remaining sections either elaborate on facets of Part II or offer answers to questions likely to be raised by those serious about establishing a quantitative literacy program.

The Dynamics of Quantitative Literacy

According to Alan Schoenfeld, mathematics educators feel they now have "a well-established theoretical frame for the characterization of intellectual competencies in problem-solving domains." (See *Entry-Level Undergraduate Courses in Science, Mathematics and Engineering: An Investment in Human Resources*, a workshop report published by Sigma Xi in 1990.) For this frame he lists five aspects of intellectual competency:

1. resources;

2. problem-solving strategies or heuristics;
3. control;
4. beliefs about mathematics and problem solving; and
5. practices.

The term "resources" includes knowledge of concepts, facts, and procedures, whereas "problem- solving strategies" encompasses the ideas expressed in George Polya's book *How to Solve It*. "Control" involves knowing how and when to use the resources and strategies in a manner that is both effective and efficient. And "practices" means, in the words of Lauren Resnick, "acquiring the habits and dispositions of interpretations and sense-making."

In the same paper Resnick goes on to say "...we may do well to conceive of mathematics education less as an instructional process (in the traditional sense of teaching specific, well-defined skills or items of knowledge), than as a socialization process. In this conception, people develop points of view and behavior patterns associated with gender roles, ethnic and familial cultures, and other socially defined traits. When we describe the processes by which children are socialized into these cultural patterns of thought, affect, and action, we describe long-term patterns of interaction and engagement in a social environment, not a series of lessons in how to behave or what to say on particular occasions. If we want students to treat mathematics as an ill-structured discipline-- making sense of it, arguing about it, and creating it, rather than merely doing it according to prescribed rules--we will have to socialize them as much as to instruct them. This means that we cannot expect any brief or encapsulated program on problem solving to do the job. Instead we must seek the kind of long-term engagement in mathematical thinking that the concept of socialization implies."

In this spirit quantitative literacy for college students is not something gained by taking one specific course in the curriculum, by learning some specific mathematical content, or by developing a particular level of computational facility. Becoming quantitatively literate must not be thought of as acquiring a certain set of concepts, facts, and procedures (through a remedial course, if necessary) followed by completion of a good course in problem solving. Further, becoming quantitatively literate is not the consequence of an excellent survey course (or introductory statistics sequence) which does no more than engage the student in doing interesting application problems. Rather, a student becomes quantitatively literate through a broad program aimed at developing capabilities in thought, analysis, and perspective--through a program aimed at developing the ingredients expressed in Part II of this report in such a manner that the student will have formed attitudes and habits of thought which provide certain "long-term patterns of interaction and engagement."

Entry Points to College and the Attainment of Quantitative Literacy

In order for a college to have a well-defined program through which a student may become quantitatively literate, the institution must pay careful attention to the critical transitions students undergo. As preparation for the college literacy requirements, college-bound high school students should be encouraged to take as many years of mathematics as their schedules allow, and especially to take mathematics during their senior year.

Four-year colleges and universities ordinarily dictate standards for acceptance. For example, two or more years of college preparatory high school mathematics may be required for admission, whether the institution be public or private. Clearly, any program for the development of quantitative literacy must take into account the level of mathematical attainment at which the student enters and seeks a smooth transition for the student. Further, in order to maintain a quantitative literacy program of high quality, four-year colleges and universities should expect transfer students to have made the same amount of progress toward quantitative literacy as native students. Performance on a good placement test (covering computational facility, mathematical reasoning, and problem solving) should be immensely helpful to advisors in determining where a student enters the quantitative literacy program.

When the degree of quantitative literacy expected for all college graduates was described earlier, it was described as more than a matter of remediation. However, one or more remediation stages may be needed in a quantitative literacy program.

Mathematics Across the Curriculum

While colleges and universities should strive to ensure that every baccalaureate recipient has achieved \ql , departments of mathematics must accept responsibility for providing leadership in establishing a focused quantitative literacy program within their institutions and seeing that it is maintained in a suitable manner. Such a program has a parallel in the "writing across the curriculum" programs which have emerged on college campuses in the last 15 years. In fact, some of the same arguments for the development of writing programs make sense when applied to the development of programs for quantitative literacy . Consider, for example, the following argument of Barbara E. Passler Walvoord for writing across the curriculum (*Helping Students Write Well--A Guide for Teachers in ALL Disciplines*, 2nd ed., Modern Language Association of American, NY, 1986, p. 4).

> *"All right," we might say, "if people are so enthusiastic about English, they should take English. Why doesn't the English department teach them how to write?" The English department can't do the whole job. Writing is so complex an activity, so closely tied to a person's intellectual development, that it must be nurtured and practiced over all the years of a student's schooling and in every curricular area. The Composition class can give students some transferable skills: the concept that effective writing is focused and well organized, strategies for structuring forms like comparison or argument, principles of clear prose style, and conventions of grammar and punctuation. Like every class, however, the composition class is a community, with its own set of expectations, types of writing, and unique values. When asked, "What is the goal of your course?" faculty members usually give a discipline-specific answer--"to make my students think like economists," or "To teach my students the basic questions, methods, and values of psychology." Instructors in those fields must show students how to apply composition skills and how to carry on the common types of thinking and writing in the individual discipline.*

Just as the complexity of the writing task is so great that the English department should not be expected to assume responsibility for the entire job of its development for the student, so also the complexity of the task of a student's becoming quantitatively literate requires the commitment of more than the department of mathematics. Instructors in other fields must show students how to *apply* quantitative reasoning to gain disciplinary knowledge and understanding.

However, in assuming a leadership role for the quantitative literacy program, the mathematics department will see to it that there is a means within the institution for students to secure a foundation in their quest for quantitative literacy . For example, depending on the student's entering computational facility level and intended major, the student may be directed to one of a set of courses which seek to advance the ingredients of quantitative literacy described in Part II of this report. For many mathematics departments this may well mean the establishment of at least one new course whose thrust is quantitative literacy as well as the reexamination (and possible alteration) of other courses in the department and college to make them better at fostering quantitative literacy goals. The foundation experience should aim to develop the student's capability to DO quantitative reasoning -- not just to see it done! But the foundation experience should not stand alone!!!

The aspects of intellectual competency termed "control," "beliefs," and "practices" in our earlier discussion of the dynamics of quantitative literacy ordinarily involve changing attitudes and habits. And new attitudes and habits take time to acquire. Consequently, the foundation for the quantitative literacy program should be obtained early in the baccalaureate career to give the student the growing time essential to the program's success. If students are to acquire quantitative literacy as a base for life-long learning and see connections between mathematics and other disciplines, they must really experience such learning and connections.

Experiencing such learning means not suppressing the use of quantitative reasoning in science and social science general education courses the student takes for the degree. It means encouraging students to carry out projects involving quantitative reasoning in courses outside the mathematics and natural science departments. Also, since quantitative literacy can be expected to be more strongly developed if it is encountered in settings students consider relevant to their interests, the curricula for all program majors should in some way include the experience of quantitative reasoning in courses at the junior and senior level. Thus, throughout their undergraduate careers students should be faced with reinforcement and strengthening of the type of thinking encountered in the foundation.

Those curriculum committees in the college and university which set the general education program for all students must be made aware of the "why" of quantitative literacy . That awareness can lead to the formation of a network of faculty, staff, and administrators who can develop a plan for a program and engender the good will, hard work, and cooperation needed to bring the program into being. In addition, a focused program should involve the establishment of explicit targets of accomplishment for the student which can be meaningfully assessed.

In a successful quantitative literacy program faculty members will have a role as coaches who again and again model quantitative reasoning and who critique student performance so that it may be recognized as good when it is and be improved or further developed when it is not so good. The work of these faculty coaches will be enhanced by the presence of a supportive learning environment for quantitative literacy on the whole campus.

Across the campus, faculty (especially those serving as advisers) should act as cheerleaders for students acquiring quantitative literacy by exuding the value of the experience. Supportive learning environments counteract anxious and phobic responses to quantitative reasoning situations (see S. Tobias and C. Weissbroad, "Anxiety and math: an update," *Harvard Educational Review* (1980), and Anne Wescott Dodd, "Insights from a Math Phobic," *Math. Teacher* (1992). Establishment of a mathematics clinic paralleling the writing clinic could be of immerse value too! (In fact, verbalizing or writing questions arising in a quantitative literacy setting may help a student develop multiple communication skills.)

New Courses and Course Materials

Although it need not be, the foundation experience for many students in a focused quantitative literacy program may well be a course taken in the mathematics department. However, it is likely to be different from existing courses. It should be clear from Part II of this report that the standard intermediate algebra and college algebra courses are generally not of the nature proposed. Also not of this nature are courses surveying selected topics whose only goals are to expose students to mathematical beauty and power. (Of course, this means that CLEP examinations and other similar examinations aimed at showing proficiency for one of these courses are also not sufficient to assess quantitative literacy.) In short, new courses and course materials need to be developed-- courses which are designed to take students at the entry levels of the institution and immerse them in doing quantitative reasoning of a nonroutine nature as described earlier. Some such courses are emerging at liberal arts colleges and elsewhere--for example, at schools receiving support for the New Liberal Arts Program of the Alfred Sloan Foundation: Mt. Holyoke College, the University of Chicago, and SUNY at Stony Brook (see S. Goldberg, ed., *The New Liberal Arts Program; A 1990 Report*, (1990)).

The development of courses and course materials is normally a demanding activity, but for a foundation course for a quantitative literacy program, it is especially demanding. Writing in September 1990 about the development of their course first offered in the 1987-88 academic year at the University of Chicago, J. Cowan, S. Kurtz, and R. Thisted explained:

> “ *When we set out to develop this course, we expected to do research, to write books, to write programs that supported the books we were going to write, and in short to erect a pedagogical monument to mathematical thought within the liberal arts out of nothing but our own will. Such hubris!* Of course, lofty goals

> *like this don't just die; and we weren't being hypocritical in holding them. We persevered in this program; we wrote essays (not quite yet books); we wrote programs (but without all the polish we would have liked); we've taught (and continue to teach) a good course that is getting better. The problem with this approach is that it has not been efficient. ...Our major objective has been to produce an exportable course. ...We've had considerable discussion about what "exporting" means; but we're barely to the point of getting our colleagues to teach the course here, let alone getting the course taught at other universities. ...We still intend to produce the materials to export the course. Indeed, we have been working and continue to work. There is no doubt that we underestimated the effort necessary to get the job done; but our resolve to do it is intact.* ”

(From "A course in the Mathematical Sciences," in S. Golberg (ed.), *The New Liberal Arts Program; A 1990 Report*)

Course materials for a foundation course must place emphasis on students' doing reasoning, rather than merely being exposed to it. Materials should capture student interest, which may well mean that they are dated in nature (of a "throw away" character). Topics studied should have genuine application (there are plenty of mathematical topics with both utility and beauty, so beauty need not be sacrificed). Exercises and problems generally must be better problems or exercises, not just harder exercises or problems than those which have appeared frequently in mathematics courses aimed at increasing mathematical competency. The "natural" use of hand calculators and appropriate computer software should be involved. And materials must tackle inappropriate "beliefs" that students may carry, such as "to do mathematics is to calculate answers." Further, materials should be usable for teaching methods different from the lecture and listen mode.

Producing projects or laboratory exercises or computer programs for courses may take considerable time and energy. For example, in the University of Chicago development experience the faculty wrote the following about a computer program they wrote to simulate the propagation of a plague: "This program required about a month of evenings to write, and provides a 10 minute demonstration for the class together with a single homework assignment."

One source for materials is the Consortium for Mathematics and its Applications (COMAP), Inc. Some of its UMAP modules have the characteristics of the course materials sought for a foundation course. Although they are written for grade levels 8-12, the American Statistical Association's Quantitative Literacy Series presents materials of the nature we are suggesting.

In order for a high-quality and focused quantitative literacy program to be established at a college, the college needs to support faculty who are willing to set up the program--which may mean developing new courses and course materials in the mathematics department or elsewhere in the institution. Further, attending development workshops, perhaps in periods when classes are not in session, may help faculty to be prepared to participate in the quantitative literacy program at all levels. The common set of goals for the institution's program and the means for their attainment need to be understood well by all those faculty who provide the program. In particular, all faculty should know the base the foundation course seeks to establish, so the follow-up components can reinforce and firmly build on that base. In order to be effective the quantitative literacy program must be "bought into" by the faculty in a college as a whole and the curriculum must reflect the belief that quantitative reasoning belongs in courses outside departments of mathematics.

Connections with Existing Courses

Could any existing courses in a college or university experience serve as the foundation experience for a focused quantitative literacy program? The critical test of whether a course could is not where it is housed in the college or university (it might be an interdisciplinary course or one in a technology department or one in a mathematics department), but how well it meets the conditions given in Part II. A quantitative literacy program may have many means for obtaining the foundation experience, thereby allowing for variations in a student's mathematical accomplishments and intended program of study. Further, the foundation experience may take on a variety of forms even when it consists of courses: it may consist of more than one course, or be one course for some students and more than one for others. But however it is set, *all* college students should have an experience with the ingredients stated in Part II linked in a natural way with additional courses, laboratories, projects, proficiency demonstrations, or whatever is needed for that individual student to complete the quantitative literacy program. In order for the course offerings of a mathematics department to fit well with an institution's quantitative literacy program (not just the mathematics department's program) the department may need to make some modification in content, emphasis, teaching methods, configuration, and credit hours awarded for its entry-level courses. Such modifications will demand careful articulation with other programs, such as the program for prospective elementary school teachers or the baccalaureate transfer curricula for two-year college students.

Means of Teaching Quantitative Literacy

Part II has suggested that remedial courses in mathematics should be taught "subversively." In the MAA Report "Minimal Mathematical Competencies for College Graduates", we read:

> “*Students entering college with mathematical deficiencies have presumably had opportunities to learn the mathematics, and for them those opportunities did not work. Therefore, the college remedial course should not be a mere rehash, and certainly not an accelerated one, of the traditional secondary or even elementary course. Courses that cover the same old ground in much the same old way tend to be just as uninspiring and unintelligible for these students as the originals, and therefore even less likely to succeed. Students should be able to find even remedial courses fresh, interesting, and significant.*”

For such remedial work the "fresh, interesting, and significant" approach we advocate is to study the mathematics in context. As expressed in the report "Reaching for Quantitative Literacy" (in *Heeding the Call for Change: Suggestions for Curricular Action*, (1992)), "The key is to have the context relate to student interest, daily life, and likely work settings." Or as also stated there, teaching approaches should provide relevance to the student's life as the student perceives it. As anyone who reads college student newspapers knows, students *can* be interested in problems they may genuinely face as citizens in our communities, states, and the nation.

For nonremedial quantitative literacy courses both to capture student interest and to enable students to make connections with other disciplines, a wide variety of issues should be discussed in problem settings. Citizens who are leaders in their communities will need to respond to issues of personal finance, the environment, personal health, politics, the local schools, and the like. General education courses in the sciences and social sciences commonly raise issues which may be discussed in quantitative terms--a good quantitative literacy program will encourage them to do so.

But what approaches can be used to stimulate student learning of mathematics in context? The report *Everybody Counts* notes that educational research as to how students learn points to the need for teachers to view their roles as broader than they may have in the past. On page 58 and following we find:

> “*...Educational research offers compelling evidence that students learn mathematics well only when they construct their own mathematical understanding... All students engage in a great deal of invention as they learn mathematics; they impose their own interpretation on what is presented to create a theory that makes sense to them. Students do not learn simply a subset of what they have been shown. Instead, they use new information to modify their prior beliefs. As a consequence, each student's knowledge of mathematics is uniquely personal. Evidence that students construct a hierarchy of understanding through processes of assimilation and accommodation with prior belief is not new; hints can be found in the work of Piaget over fifty years ago. Insights from contemporary cognitive science help confirm these earlier observations by establishing a theoretical framework based on evidence from many fields of study. No teaching can be effective if it does not respond to students' prior ideas. Teachers need to listen as much as they need to speak. They need to resist the temptation to control classroom ideas*

so that students can gain a sense of ownership over what they are learning. Doing this requires genuine give-and-take in the mathematics classroom, both among students and between students and teachers. ... Teachers' roles should include those of consultant, moderator, and interlocutor, not just presenter and authority. Classroom activities must encourage students to express their approaches, both orally and in writing. Students must engage mathematics as a human activity; they must learn to work cooperatively in small teams to solve problems as well as to argue convincingly for their approach amid conflicting ideas and strategies. ...As students begin to take responsibility for their own work, they will learn how to learn as well as what to learn. ”

Other research that is especially pertinent to the goals of quantitative literacy are studies on how to teach problem solving. In an essay "Teaching Mathematical Problem Solving: Insights from Teachers and Tutors" (1989)), R. Shavelson, N. Webb, C. Stasz, and D. McArthur studied expert teachers and tutors of the process and compiled what they believe to be some important features of successfully teaching problem solving. These features include:

1. Activating students' prior knowledge relevant to teaching a new concept.
2. Using multiple representations to teach a mathematical concept.
3. Coordinating and translating among alternative representations so that students see a concept in multiple ways.
4. Evaluating problem-solving performance in an ongoing manner.
5. Providing informal proofs.
6. Providing detailed explanations and justifications of reasoning in problem solving.
7. Using specific examples to illustrate a general concept.
8. Tuning in to students' problem-solving processes so that errors and gaps in understanding can be corrected immediately.

While acknowledging that this list is incomplete, the essay goes on to observe that the features do not appear individually in teachers and tutors, but they collectively become well integrated in the working of excellent teachers and tutors. Taken as a whole, the features suggest strategies for teaching which are more interactive in character than the traditional lecture and listen mode.

There are methods for teaching mathematics courses aimed at quantitative literacy which have the added benefit of blending well with succeeding components in a quantitative literacy program. We advocate the following examples: establishing collaborative learning situations, utilizing a wide variety of writing assignments, studying significant mathematical models, conducting explorations using calculators or computers, and employing team projects.

A description of the nature of collaborative learning situations appears in the report of the Sigma Xi workshop cited earlier.

“ *During the workshop, Jack Lochhead demonstrated collaborative learning by involving all participants, working in groups of two, in the solution of simple problems. It became immediately evident that the work could be structured in such a way that each participant has the experience of formulating and articulating questions, and the experience of formulating and articulating responses to questions. The ground rules were very simple. One of the pair assumed the roles of reading and answering questions, talking aloud throughout; the other assumed the role of asking questions. With the next problem the roles reversed. What a wonderful way for individuals to explore problem solving and at the same time explore their own mental constructs and confront misconceptions in a friendly environment.*”

> *"Uri Treisman, reports success with students considered to be at risk in entry-level calculus by combining work in small groups with traditional lectures. The students register for the regular calculus course and also for a scheduled laboratory that meets for a two hour session twice a week. These laboratories are devoted to the investigation of problem solving with the students working in small groups under the supervision of a competent mathematician who understands how to select appropriate problems and assist small groups in their investigations of problem solving. This extension of time-on-task, working in small groups under supervision, enables students to have experience in doing mathematics and to recoup in part the deficit in problem solving experienced during the precollege years. The essential characteristics of collaborative learning are scheduled periods of working in small groups with other students on common problems in a supportive environment (p.12)."*

A second method for teaching for quantitative literacy is to employ writing. A writing assignment may be simple: asking students to explain problem results, or asking them to develop lecture notes for a class period. More complex assignments are reporting on a team project or a discovery laboratory experience or making a critical analysis of a media presentation of quantitative information. Asking a student to write an explanation of problem results demands that the student clarify his/her thinking about the problem and how it can be solved. Further, it enables the faculty member reading the student's paper to respond more pointedly to the student's thought processes and hence be more helpful. Two excellent sources for help with implementing writing as a teaching tool are the MAA Notes Number 16, *Using Writing to Teach Mathematics* (edited by Andrew Sterrett), and the collection of essays *Writing to Learn Mathematics and Sciences* edited by Paul Connolly and Teresa Vilardi and published in 1989 by Teachers College Press of Columbia University. A review of these sources and descriptions of the quantitative reasoning courses developed with the help of the Sloan Foundation reveals substantial linkage between the use of writing as a means to quantitative literacy and the other methods suggested above. For example, all three of the courses mentioned earlier, which the Sloan Foundation helped develop, require students to write papers in response to questions about significant mathematical models.

Many students will encounter quantitative reasoning in their major programs of study through consideration of significant mathematical models. As noted in the MAA report *A Call for Change* (p. 5): "There are three ways in which models are used in attempts to solve problems originating in the world around us: (i) information is derived from mathematical models that others have built and used (weather reports, governmental studies, stock market projections); (ii) models that others have derived are used to analyze real-world situations; and (iii) people derive their own mathematical models of the situations either from known quantitative relationships or from collected data." All three of these ways may be brought into a course on quantitative methods by making a careful choice of models to consider. At the same time the choice can be made so as to study mathematics which is entry-level for college students.

Some mathematical models may be set up as computer-based exploratory environments in which the student may change parameters and recognize relationships in a model by "playing" with it on the computer. Such explorations may increase students' mathematical power. As stated in *A Call for Change* (pp. 6-7), "Greater accessibility to mathematical strategies and representations provided by a technologically rich environment opens a new array of real-world problems to mathematical solutions. Calculators and computers allow students to explore mathematical ideas from several different perspectives, resulting in a deeper mathematical understanding. Through the regular use of calculators and computers in collegiate mathematics courses, students learn more mathematics and can more rapidly apply that understanding in problem solving." In particular, calculators and computers may be used "to pose problems, explore patterns, test conjectures, conduct simulations, and organize and represent data" (ibid., p. 7). But also, perhaps in a more mundane view, employing calculators and computers may "force" students to come to grips with the order of operations involved in elementary computations such as those involved in determining the standard deviation of a data set. In the latter instance the use of technology may create a need to know something the student has not learned previously, but which the student is now willing to pursue seriously.

Computer-based or calculator-based explorations need not appear in courses as "exercises" but may be solidly integrated into student team projects. Projects assigned to student teams may involve the exploration of a model simulated by a computer program or the carrying out of activities more routine in nature, such as conducting a poll. The key to their development is that they involve mathematical ideas in context. Such projects would be expected to result in written, and perhaps oral, presentation of results obtained.

The methods for teaching mathematics courses aimed at quantitative literacy discussed here may be used in various proportions within a course and may be used in combination with each other as the paragraph on joint projects suggests. These methods not only can mesh well with the later components of a quantitative literacy program, and incorporate important features of teaching problem solving, but they also may provide the needed care which a successful transition from students' earlier experiences with mathematics may require, as well as a means for attacking the false beliefs about mathematics or the anxious or phobic responses to mathematics that some students exhibit.

Indeed, many myths about mathematics and the ability to do it have been propagated in our society. Activity-based learning methods cut through the alienation some feel towards mathematics and reach out to those who have until now seen mathematics as a "solitary" or "purely intellectual" endeavor. Cooperative learning where students encounter problems they must discuss with other students helps breed success for those who may not have previously known great success in quantitative reasoning situations -- thus changing students' perceptions of their own ability. Further, using team projects and studying a variety of mathematical models may lead to realizations that mathematics is not only for the few, nor is it a numbing experience (in fact, it could become enjoyable!). Working with a team of students may lead an individual student to a sense of community in a class (a desirable trait for liking a class, as pointed out in S. Tobias, *They're Not Dumb, They're Different*, p. 22) and also to the recognition that there may be more than one way to do a problem. Cooperative learning creates an atmosphere of acceptance and openness for those who may have known mathematics anxiety or phobia in the past, helping develop at least a "neutral" attitude towards mathematics, if not a positive one. And using calculators and/or computers for explorations may present mathematics as not just computation. In general, those methods suggested above should also counteract the types of feelings expressed by students such as Eric towards the teaching style of his physics class as reported in Tobias (ibid; p. 21):

> *“I still get the feeling that unlike a humanities course, here the professor is the keeper of the information, the one who knows all the answers. This does little to propagate discussion or dissent. The professor does examples the "right way" and we are to mimic this as accurately as possible. Our opinions are not valued, especially since there is only one right answer, and at this level, usually only one (right) way to get it.”*

Undergoing transition from the type of teaching method Eric encountered to the types suggested above for courses aimed at quantitative literacy demands that students be given special attention. If students are to become confident in their ability to analyze, discuss, and use quantitative information, they must encounter sufficient practice and success in using quantitative information. Students must be expected to do quantitative reasoning--reasoning which is substantially different from completing routine problems. Where previous student experience has been largely routine in character, students may need to be "phased in" to teaching methods which present greater and different demands on them. Explaining to students why they are being asked to do certain activities often garners both greater acceptance of the activity by the students and more effort in carrying out that activity. Further identifiable goals need to be articulated to students, and students need to be advised on how to study, what are reasonable expectations for themselves, and how to use the available resources (including faculty office time).

Finally, the ways used to evaluate mathematics courses aimed at quantitative literacy should reflect the course goals and the teaching methods used. For example, evaluation components may include written assignments, participation in discussions, and examinations. And examination questions may involve routine questions along with discussion and open-ended questions; in short, students should be expected to respond to "why" questions and not just "how" questions and to show that they know how to raise pertinent questions.

Two excellent sources for additional ideas and elaboration of some of these ideas are the essay "Teaching Statistics" in the MAA volume *Heeding the Call for Change* and the MAA publication *A Sourcebook for College Mathematics Teaching*.

Key to Opportunity--Impact on Women, Minorities, and the Disadvantaged

A focused quantitative literacy program must not be viewed as one more "hurdle" for underrepresented groups in our culture. In particular, while the program must be carefully developed, it must also be sensitively executed. Periodic assessment of student attitudes and needs may make desirable adjustments possible and ensure the cooperative spirit of students.

Ingrained beliefs die hard. So women and ethnic group members who have long been told, often in subtle ways, that they are not capable or are less capable of doing mathematics may need repeated encouragement to do the hard work on which success depends. Students who are labeled "at risk" may find themselves fighting anxious and phobic responses and be in need of special care. The formation of study groups in the execution of teaching strategies should be done in full awareness of the special needs groups of students may have.

Probably the most crucial time in the quantitative literacy program for those who come from underrepresented groups is the foundation experience. Thus, critical to success in the program is the advice resulting in the initial placement of the student as well as the help and encouragement received in that initial experience.

Factors in Establishing and Maintaining a Program

While it is acknowledged that the establishment of a quantitative literacy program at any institution is likely to take considerable energy on the part of the mathematics department, the climate is ripe to move on such a program now!

The desire to improve opportunities for certain groups to participate more fully in our society, the actions that the government and the mathematics community are taking to improve the standards of school mathematics, and the pressures on colleges and universities for accountability regarding undergraduate education all provide avenues for change in core programs for all baccalaureate students. The accountability pressures alone can lead to a natural reexamination of the extent to which graduates are quantitatively literate, but the establishment of a program of writing across the curriculum also provides an ideal time to advance a program of quantitative literacy. The latter is true because then the parallel between mathematics across the curriculum and writing across the curriculum can be more easily grasped. Also it is easier for a faculty and others to institute changes of the magnitude suggested in a package than to try to adopt one major change followed by another.

As noted indirectly in the discussion of mathematics across the curriculum, the establishment of a quantitative literacy program will take a network of dedicated faculty and others. Users of mathematics on campus should be actively involved in setting the program. While the mathematics department should be expected to bring leadership to the establishment of a program and its standards, and play a prominent ongoing role in its execution, a quantitative literacy program can normally be expected to be overseen by a general education committee of the college. To be effective, such a committee must accept responsibility for advocating quantitative literacy for all students as well as being a support group for those faculty striving in the front lines for student development.

A quantitative literacy program can most easily be implemented in stages and should be expected to take time to be fully executed. Having made a plan, a committee may first introduce foundations experiences along with appropriate placement processes for entry into those experiences. Later the continuation experiences can be added to the program or defined for the program. What is important is to *GET STARTED* on a quantitative literacy program, and, from a practical standpoint, to realize that the program can be phased in.

It should be noted that it is not being suggested that the concept be implemented in an artificial way--e.g. by asking art majors to take irrelevant mathematics courses in their junior and senior years. Rather some students may be taking general education courses which have a substantial quantitative component (presupposing the foundation experience) during their junior and senior years in college. Or some students may be completing research methods courses in disciplines outside mathematics, or doing substantial projects in their coursework which involve quantitative reasoning.

Some computer science courses may be acceptable quantitative reasoning experiences, while others will do little to foster the goals of Part II. But computer literacy and quantitative literacy are *not* the same. Generally computer literacy aims to enable a student to use a personal computer effectively in word processing and provides no experience in quantitative reasoning.

Stages of Quantitative Literacy and Outline of a Program

The starting point for any program in quantitative literacy will ordinarily be the college entrance requirements. These requirements and an appropriate related placement test established on both the goals of quantitative literacy and background needs for courses for various majors and degree programs should determine the foundation experience to which the student is directed. In many colleges there will be multiple tracks in the quantitative literacy program, but each will normally have the following components:

1. Explicit requirements of quantitative experience for college entry or for entry into courses or experiences which can be credited towards the baccalaureate degree;
2. Placement testing intended to help determine appropriate entry into the quantitative literacy program;
3. Foundation experience(s) to be accomplished ordinarily within the first year of the student's college work;
4. Further quantitative experiences in diverse contexts to be accomplished during a student's sophomore, junior, and senior college years so as to be interspersed throughout the work of these years.

The components will take on a variety of expressions. To clarify our vision we offer the following hypothetical examples of possible programs:

Example 1. Students are admitted with three or four years of specified college preparatory high school mathematics courses and sufficiently high ACT or SAT scores. Guided by a testing program and student interest, each student takes general education courses that require quantitative reasoning projects at least one of which employs statistical methods and all of which involve mathematical models. Students individually contract for additional experiences--some take courses while others do some research or write a junior paper and a senior thesis.

Example 2. Students admitted to the college have three or four years of specified college preparatory high school mathematics courses and sufficiently high ACT or SAT scores combined with high class rankings at graduation. A testing program and student interest determine whether students take an interdisciplinary course in quantitative reasoning, a precalculus course, or a calculus course--these are to be (part of) the foundation experience and are to be formulated and taught in such a way as to foster the goals in Part II. In each subsequent academic year each student is required to submit in at least one course a paper which includes a substantial quantitative component furthering attainment of the goals of quantitative literacy (as in Part II).

Example 3. A college has an entrance requirement of two years of college preparatory high school mathematics including one year of algebra. A placement test and student interest direct students into one of five courses termed remedial, competency, mathematics for social science or business, precalculus, and calculus. The course chosen is (part of) the foundation experience for the student and is to be formulated and taught in such a way as to foster the goals in Part II. Before the junior year, students are subsequently to take one course designated "Q" (see below) in the social science or science section of the college's general education program. In their junior and senior years students must take two additional upper division "Q" courses in different semesters. A "Q" course must have a substantial quantitative component in some form which builds on the foundation experience and furthers the quantitative literacy goals (in Part II).

Example 4. The college has no explicit entrance requirement in mathematics (presupposes only high school graduation mathematical experience). Admission to the foundation quantitative literacy courses presupposes knowledge of intermediate algebra. Two explicit remedial courses are taught according to the philosophy espoused in Part II--one aims at the student who has not studied mathematics for a long period of time, while the other is intended for those who have studied mathematics more recently, but have not yet attained the level of completion of an intermediate algebra course. One or both remedial courses which emphasize collaborative learning and project assignments are prescribed as prerequisite for the foundation experience for the quantitative literacy program for those students who have not yet mastered intermediate algebra. For those with intermediate algebra as background, a testing program provides guidance for placement into one of six courses termed remedial, quantitative reasoning I, finite mathematics, mathematics for elementary teachers, precalculus, and calculus. All six courses are formulated and taught in such a way as to foster the goals in Part II. In their junior and senior years students must complete two additional courses from a quantitative reasoning listing of courses.

However the program is formulated, targets for student accomplishment through the foundation experience should be set, and the entire program should be directed towards the goals of Part II. In any case, the program should have an assessment component which can be used to improve the program.

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PART IV: Assessment

The establishment of a quantitative literacy program is expected to go hand-in-hand with an attendant assessment procedure. By assessment procedure we mean an explicit means for obtaining information on the program's impact on student development. This information can then suggest modifications in the program which may lead to enhanced teaching and learning. Assessment is a mechanism to guide and encourage improvement. Thus assessment procedures may evaluate individual components of the quantitative literacy program as well as the program as a whole.

There are many problems involved in assessment in academia in general. Fears abound that assessment efforts will lead to the exposure of glaring weaknesses in our educational process or clearly identify individual or group shortcomings which can be moved into political arenas where such findings may be abused. Further, our society's understanding of the role of mathematics and its content may differ widely from that in the academic community. However, open discussions in the media and leadership at the national level are working to improve understanding and foster systematic and convergent change. The diversity within our society makes it critical that the mathematics community act in a cohesive and responsible fashion. Thus conducting and acting on assessment procedures should be a normal part of our activity.

Assessment must be sensitive to reality. The recently announced new national goals for science and mathematics represent a lifting of expectations in a time of changing demographics, technologies, and global challenge. The problems of background, equity, accessibility, cost, ethnicity, and gender are important considerations in making an assessment, but they do not determine standards for student performance. Students with weak backgrounds and students from underrepresented groups in higher education do not need different standards, but they need effective programs through which they can reach appropriate standards. Carefully constructed assessment procedures should seek to measure the effectiveness a program has for *all* the students in it.

Assessment should be based on what we understand about how students learn. It is more than mere testing as commonly used in classrooms, because it seeks to dissect the components of student learning rather than simply evaluate final performance on mathematical tasks. The section on the dynamics of quantitative literacy in Part III suggests five aspects of intellectual competency which need to be considered. Efforts should be made to determine how effective a quantitative literacy program is in fostering the development of each of these aspects.

Assessment should fit the nature of a quantitative literacy program using methods which reflect the type of learning to be measured, rather than methods which are most easily constructed or scored. For example, if students are to learn how to respond to open-ended problem settings, they must be asked to do so in the assessment procedure. Facing them with a multiple-choice test for the measurement of such a goal would be inappropriate.

Assessment should be an integral part of the teaching-learning process and not an add-on. Students learn from good assessment methods what they are being asked to learn while teachers obtain feedback on not only what students have learned, but how students have learned it or aspects of learning which need further attention. Thus assessment is carried on throughout the students' progress through the quantitative literacy program to inform the teaching and learning rather than being a measure of student accomplishment at the end of a program cycle alone.

Another principle on which assessment is based for quantitative literacy programs relates to the interdisciplinary character of such programs. Since quantitative literacy programs are expected to be interdisciplinary in character, assessment methods should reflect that intent--clear applications-oriented tasks should be present in the methods.

Assessment should also provide opportunities for students to demonstrate what they know and can do in a variety of ways. Offering students a variety of ways to demonstrate learning ensures that a measurement can capture what students have really learned rather than that they can respond to a particular type of evaluation method. Further, a variety of approaches can better capture how students construct knowledge and match with student experiences and needs.

Developing a valid assessment procedure may be considered to be very costly in both faculty time and money. We recommend, nonetheless, at least a minimal program aimed at the extent to which the quantitative literacy program helps students reach the goals (1)-(5) in Part II.

An outline of a process to follow in assessment of either the total quantitative literacy program or its components follows:

1. Review the goals as set forth in (1)-(5) in Part II which the quantitative literacy program seeks to help students accomplish.
2. Review the instructional strategies which resulted in the design of the quantitative literacy program--how the components and the total program are set to foster (1)-(5) in Part II for students.
3. Review the performance standards which have been openly developed and communicated in linking the goals and the strategies.
4. Choose assessment methods to measure student learning resulting from the instructional strategies.
5. Once assessment methods have been executed, summarize what is working, what is not working, and what could be working better in the learning-teaching match. To do so compare data obtained through the assessment with the goals and preset performance standards. Make comparisons over time also.
6. Determine changes in courses, experiences, or placement processes, or in the program as a whole which can be implemented to lead to a more effective learning and teaching match. Determine changes in assessment methods which can better measure what is being learned and taught.
7. Institute changes and begin the cycle again.

In order to make gathering of information on program functioning both efficient and less costly, we recommend that as far as possible advising and program impact data be gathered simultaneously. For example, we suggest that a diagnostic placement test which includes sections on computational facility, mathematical reasoning, and elementary problem solving be administered to all entering students; then test results may be used along with other student data to determine student placement in the quantitative literacy program foundation experience as well as to establish a base line data set from which program achievement may be measured. Further, we suggest that at the same time a placement test is administered a brief attitude survey be given and its results used in the dual fashion mentioned.

We recommend that assessment data be gathered at least at two other times in the student's progression through the program--at the completion of the student's foundation experience and within a semester of the student's degree program completion. A meaningful sample of the data gathered should be analyzed each year by an appropriate faculty committee charged with making recommendations for actions which lead to program improvement.

Where it is feasible, we recommend that a senior assessment study be made which seeks to measure attainment of the goals (1)- (5) of Part II by a process of the following nature:

> “*(i)The main ingredient in the assessment should be a project where the objective is to answer certain questions and to draw conclusions on the basis of presented data. Students would be expected to analyze the data using whatever mathematical or statistical tools are appropriate, and then make those predictions and conclusions that seem reasonable. Students should be expected to conclude with a summary of the project in writing. While the project should be long enough to serve its purpose, it is not expected to be more than something which might be completed in a day or two.*”

> “*(ii)In addition to the project, the assessment should contain a variety of short answer questions related to the goals (1)-(5), formulated in such a way that students are required to explain their answers and justify their conclusions. (iii)To the extent possible, questions should be presented in a problem context so that students must choose which tools are appropriate to solve the problem and then in the course of the solution demonstrate that they can use these tools.*”

In instances where upper division courses are used to complete the quantitative literacy program, those faculty who teach such courses might be asked for a judgment of student success in meeting goals (1)-(5). Scoring guides with samples of student work at different levels of quality can be provided to guide the judgment process.

Other possible methods for obtaining assessment data include gathering student portfolios, studying student journal writings, interviewing random groupings of students, conducting group examinations, and requiring senior oral presentations or examinations.

Whatever the assessment process, it should be subject to periodic review also. Hence an ongoing board or committee should coordinate assessment activities and interact regularly with all groups responsible for the quantitative literacy program.

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Appendix A: An Annotated Bibliography

[This is a *selective* bibliography, listing only items cited in the body of this report and a few others, mostly recent. Many of these items contain useful further lists of references to the literature. The acronyms MAA, NCTM, and NRC stand respectively for the Mathematical Association of America, the National Council of Teachers of Mathematics, and the National Research Council.]

1954 Summer Writing Group of the Department of Mathematics, University of Kansas, *Universal Mathematics, Part I: Functions and Limits*. Lawrence, KS: University of Kansas, Student Union Book Store, 1954.

> *"The first half of the experimental and preliminary general mathematics text for first-year college students produced at the behest of the MAA Committee on the Undergraduate Program (later to become CUPM). The second part is listed below under the name of R.L. Davis."*

Bell, Max S., "What does Everyman' Really Need from School Mathematics?" The Mathematics Teacher 67:3 (March, 1974) 196-202. (Reprinted 87:7 (Oct., 1994) 546-551).

> *"This paper answers the question in its title with a list of topics and offers some advice about how the list might be used."*

Cheney, Lynn V., *50 hours: A Core Curriculum for College Students*. Washington, DC: National Endowment for the Humanities, 1989.

> *"A broad and demanding prescription for general undergraduate education (a "core") that puts more emphasis on mathematics than might have been expected. A few examples are given."*

Connolly, Paul, and Vilardi, Teresa, eds., *Writing to Learn Mathematics and Sciences.* New York: Teachers College Press, Columbia University, 1989.

> *"An excellent collection of essays on the subject."*

Committee on Support of Research in the Mathematical Sciences (COSRIMS) of the NRC, *The Mathematics Sciences: A Report.* Washington, DC: National Academy of Sciences, 1968.

> *"One of the earlier harbingers of problems to come, but not lavish with ideas about what quantitative literacy is or how it should be achieved."*

CUPM, *A General Curriculum in Mathematics for Colleges*. Berkeley, CA:MAA, 1965.

> *"A celebrated and influential attempt to provide a curriculum that would enable a small college mathematics faculty to meet the needs of its various students. It does not definitively confront the quantitative literacy question, but on pp. 25-26 offers some interesting ideas on the general issue."*

CUPM Panel, "Minimal Mathematical Competencies for College Graduates." American Mathematical Monthly 89:4 (April 1982) 266-272; reprinted in Lynn Arthur Steen, ed., *Reshaping College Mathematics* (MAA Notes Number 13). Washington, DC: MAA, 1989, 103-108.

> *"Many of the ideas in this somewhat inconclusive report still have some value. The reprinted version includes a new preface."*

Davis, R.L., ed., *Universal Mathematics, Part II: Elementary Mathematics of Sets with Applications*. Charlottesville, VA: Committee on the Undergraduate Program, 1958.

> *"This is the second part of the experimental and preliminary attempt to provide, under the sponsorship of the MAA Committee on the Undergraduate Program (which evolved into CUPM) a text in mathematics for all "normally" prepared first-year college students. The first part is listed above under "1954 Summer Writing Group...""*

Dodd, Anne Wescott, "Insights from a Math Phobic." *The Mathematics Teacher* 85:4 (1992) 296-298.

"On the basis of first- hand experience, the author maintains that "math phobics" can become "math fans," but that nontraditional teaching methods can help."

Duren, W.L., Jr., "CUPM, the History of an Idea." *American Mathematical Monthly* 74:1, part 2 (January, 1967) 23-37.

"An absorbing chronicle of the first few years of CUPM, with information about its antecendents and suggestions for its future."

Educational Testing Service, *The Mathematics Report Card: Are We Measuring Up?* Princeton, NJ: ETS, 1988.

"A gloomy assessment based especially on the immense data resources of the ETS."

Garfunkel, Sol, ed., *For All Practical Purposes*. San Francisco, etc.: W.H. Freeman, 1988.

"A book that cultivates quantitative literacy by reliance on somewhat unorthodox applications. There exist more recent additions and supporting materials (videos, etc.) for this refreshing and widely adopted text."

Goldberg, S., ed. *The New Liberal Arts Program: A 1990 Report.* New York: Alfred P. Sloan Foundation, 1990.

"Information about the nature of the "new liberal arts" program and about activities within it. Supporters of "the new liberal arts," notably the Sloan Foundation, maintain that the mathematical sciences should be treated as important parts of a liberal education in our era."

Harrison, Anna, *Entry-Level Undergraduate Courses in Sciences, Mathematics, and Engineering: An Investment in Human Resources.* Research Triangle Park, NC: Sigma Xi, The Scientific Research Society, 1990.

"A lucid, comprehensive, and even-handed report on a workshop sponsored by the National Science Foundation and the Johnson Foundation."

Joint Committee of the MAA and NCTM, *A Source Book of Applications of School Mathematics*. Reston, VA: National Council of Teachers of Mathematics, 1980.

"Primarily a collection of exemplary problems, this book also contains some useful essays on such matters as the nature of mathematical modelling and the importance of mathematics in everyday life."

Leitzel, James R.C., ed. *A Call for Change: Recommendations for the Mathematical Preparation of Teachers of Mathematics.* Washington, DC: MAA, 1991.

"A report from the MAA Committee on the Mathematical Education of Teachers (COMET)."

Madison, Bernard L., and Hart, Therese A., *A Challenge of Numbers: People in the Mathematical Sciences.* Washington, DC: National Academy Press, 1990.

"A compilation and analysis of the data on which the other publications in the NRC series were largely based."

Mathematical Sciences Education Board, *Counting on You.* Washington, DC: National Academy Press, 1991.

"This booklet, written for nonmathematicians such as school board members, parents, administrators, etc., clearly conveys the gist of the recent NRC and

MSEB reports, most of which are listed elsewhere in this bibliography.”

Mathematical Sciences Education Board, *For Good Measure: Principles and Goals for Mathematics Assessment.* Washington, DC: National Academy Press, (1992).

“This report is based primarily on the National Summit on Mathematics Assessment (April, 1991) and contains both recommendations and generous quotations from educational and political leaders.”

NRC, *Everybody Counts: A Report to the Nation on the Future of Mathematics Education.* Washington, DC: National Academy Press, 1989.

“Offered as a "preface" to a series of publications (which have since appeared) from several arms of the NRC, this 114- page report offers a challenging discussion of the state of the whole range of mathematics education in the United States. It includes a 10-page bibliography.”

NRC, *Moving Beyond Myths: Revitalizing Undergraduate Mathematics.* Washington, DC: National Academy Press, 1991.

“The "capstone" of the efforts, as they apply to undergraduate mathematics education, leading to NRC reports of the late 1980's and early 1990's. The core of the report is an "action plan" (pages 34-42), which gives us all plenty to do.”

NCTM, *Curriculum and Evaluation Standards for School Mathematics.* Reston, VA: NCTM, 1989.

“This document, often called simply the Standards, "contains a set of standards for mathematics curricula in North American schools (K-12) and for evaluating the quality of both the curriculum and student achievement" (preface). It has gained wide influence as a base for mathematics education reform and further discussion.”

NCTM, *Professional Standards for Teaching Mathematics.* Reston, VA: NCTM, 1991.

“This complement to the preceding item contains principal sections that propose standards for teaching mathematics, for the evaluation of the teaching of mathematics, for the professional development of teachers of mathematics, and for the support and development of mathematics teachers and teaching. [A second volume complementary to the Standards, tentatively titled Assessment Standards for School Mathematics, is in preparation.]”

Resnick, Lauren. "Treating Mathematics as an Ill-structured Discipline." In Charles, R., and Silver, E., eds., *The Teaching and Assessing of Mathematical Problem Solving*, Reston, VA: NCTM, 1989.

“An essay on the advantages of not treating mathematics as a subject in which everything, including the nature of the subject itself, is settled.”

Schoenfeld, Alan, ed. *A Source Book for College Mathematics Teaching.* Washington, DC: MAA, 1990.

“An 80-page collection of useful advice, based primarily on the work of the MAA Committee on the Teaching of Undergraduate Mathematics (CTUM).”

Schoenfeld, Alan H., "Learning to Think Mathematically: Problem Solving, Metacognition, and Sense Making in Mathematics." In Grouws, Douglas A., ed., *Handbook of Research on Mathematics Teaching and Learning*, New York: Macmillan, 1992, 334-370.

“This extensive article deals with: what it means to think mathematically; the pertinent literature; and possible directions for future research on the issues. It is an excellent introduction to the field and background for further reading.”

Shavelson, R., et al. "Teaching Mathematical Problem Solving: Insights from Teachers and Tutors." In Charles, R., and Silver, E., *The Teaching and Assessing of Mathematical Problem* Solving, Reston, VA: NCTM, 1989.

Sons, Linda R., "Reaching for Quantitative Literacy." In Lynn Arthur Steen, ed., *Heeding the Call for Change* (MAA Notes Number 22), Washington, DC: MAA, 1992, 95-118.

> "*To a large extent this is a report on an e-mail focus group discussion conducted in the spring of 1991 and anticipates the present report in several ways.*"

Steen, Lynn Arthur, "Reaching for Science Literacy." Change 23:4 (July/August, 1991) 10-19.

> "*This lead article in a theme issue of a journal devoted to higher education in general paints a bleak picture of lower division teaching in the sciences and mathematics, but offers some encouraging examples of what can be done.*"

Sterrett, Andrew, *Using Writing to Teach Mathematics* (MAA Notes Number 16). Washington, DC: MAA, 1990.

> "*Thirty-one chapters, mostly reporting on actual experiences with writing as a device for learning mathematics. Full of excellent ideas.*"

Tobias, Sheila, *They're Not Dumb, They're Different: Stalking the Second Tier. Tucson, AZ: Research Corporation, 1990.*

> "*An "occasional paper" on why some students do not do so well in science.*"

Tobias, S., and Weissbroad, C. "Anxiety and Math: An Update." *Harvard Educational Review* 50 (1980) 63-70.

> "*One of several progress reports on activities aimed at reducing "math anxiety."*"

Tufte, Edward R., *The Visual Display of Quantitative Information.* Cheshire, CT: Graphics Press, 1983.

> "*A marvelous book-length essay on the methods for the graphic presentation of quantitative information.*"

Webb, Norman L., ed. *Assessment in the Mathematics Classroom* (1993 Yearbook). Reston, VA: NCTM, 1993.

> "*This well-organized collection of essays is concerned almost entirely with the precollege curriculum, and deals with the assessment much more of students than of courses or programs, but will be useful to anyone who wants to assess quantitative literacy.*"

Wolfe, Christopher R. "Quantitative Reasoning Across a College Curriculum." College Teaching 41:1 (Winter 1993) 3-9.

> "*Using concrete examples from his own experience as a professor of interdisciplinary studies, the author argues for the activity described in his title and sketches some of the possibilities.*"

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Appendix B: A Topical Listing

The list below of mathematics topics, *IF LACED WITH GOOD APPLICATIONS*, addresses all of the issues raised in Part II at least minimally.

Although the topics are displayed here according to subject matter components of traditional mathematics curricula, one should *NOT* infer that this is the best organization with which to address quantitative literacy . In fact, this kind of layer-cake organization may inhibit the very essence of quantitative literacy : encouraging multiple perspectives; informally developing intuition; and searching for connections. It may actually deepen the pitfall of just preparing for the next course. Nevertheless, the list suggests a common ground from which to begin. How much time will need to be spent on topics that the students have studied before will depend on circumstances, but deadly reviews of more or less familiar material should be avoided. (* means "less essential")

ARITHMETIC

- estimation
- percentage change
- use of calculator: rounding and truncation errors; order of operations.

GEOMETRY

- measurement: units and conversion of systems; length; area; volume.
- "familiar" shapes: rectangles, triangles, circles, cubes, cones, cylinders, spheres, the Pythagorean relationship.
- angles: slopes of lines; parallel and perpendicular lines; right angles; similarity.
- complex shapes: approximation by "familiar" shapes; solution region for a system of linear inequalities in the plane.

ALGEBRA

- linear equations: equations in one unknown; systems in two unknowns; methods of solution.
- proportionality
- graphs and tables: constructing; reading, interpreting; extrapolating from; the notions of direct and indirect variation.
- simple exponents: roots and powers; products and quotients with a common base.
- concept of function: constructing discrete and continuous functions; graphical representation of functions; zeros of functions.

STATISTICS

- experimental probability: counting; mutually exclusive and independent events.
- graphical displays of data: pie and bar charts; frequency polygons; visual impact of scale changes.
- central tendency and spread: comparison of data sets using mean, median, mode and standard deviation, quartile deviation, range; percentile rank.
- the idea of correlation: measuring and evaluating the relationship between two variables.

- common sources of error: sampling error; misinterpreting averages or probabilities; invalid comparison distributions; statistical significance; statistical "proof".
- random sampling: the count-recount technique; polls; lotteries; fair representation.
- linear fit: comparison of fit of two lines to a data set.
- quality control: the binomial distribution.
- simulation
- confidence intervals*: interval estimates; the standard error of the mean.

OTHER

- exponential change
- rates: comparison of average rates of change.
- models
- algorithms: sequential thinking; construction; relationship to formulas.
- optimization: the notions of maxima and minima of functions with or without constraints; graphical and computational methods for finding them; simple analytic methods, such as completing the square for quadratic polynomials.
- linear programming*: systems of equations in two variables with a linear objective function.
- scheduling*
- networks*

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Appendix C: Description of Some Foundations Courses

1. At **Bloomsburg State University in Pennsylvania** a foundations course presupposes only arithmetic and emphasizes the use of elementary mathematics in decision making and problem solving. Game theory is used to motivate the study of problem solving strategies. Hand calculators are standard tools. Materials written for the course have appeared as **Mathematics in Daily Life** by J. Growney published by McGraw Hill. Exercises in the book take on a variety of forms from those that require a brief application of a specific concept or method to those which require discussion or multistep application and evaluation of several ideas or procedures.
2. At **Northern Illinois University** a foundations course presupposes two years of high school mathematics including one year of algebra. The course aims to develop in the student a competency in problem solving and analysis which is helpful in personal decision- making; in evaluating concerns in the community, state, and nation; in setting and achieving goals; and in continued learning. A hand calculator is used throughout the course. The mathematical content is one-third probability and statistics and two-thirds logical statements and arguments, geometry in problem

solving, estimation and approximation (inequalities, functions, average rates of change), and general problem solving (including personal business applications). Problem sets consist of routine and nonroutine exercises. Materials written for the course have been published by Kendall/Hunt as Mathematical Thinking in a Quantitative World by L.R. Sons and P.J. Nicholls.

3. At the University of Tennessee the foundations course "Algebraic Reasoning: Motivated by Actual Problems in Personal Finance" presupposes two years of high school algebra and one year of high school geometry. The course assumes use of a hand calculator and places its emphasis "on the importance and applicability of mathematics in real life." Problems used to motivate algebraic concepts are relevant to most college students' experience. Topics covered include borrowing money to complete college, saving a lump sum for college education, consolidation of debts, periodic payments, amortization schedules and more! The course uses material developed locally by J. Harvey Carruth.
4. At the \ulUniversity of Chicago a trio of faculty have produced a series of ten-week (quarter) courses composed of short mini-courses each devoted to a single theme. Funded by the Sloan Foundation, the course development has involved the production of units such as statistical analysis of literary style, quantitative arguments and scientific method, the organization of the brain, dynamical systems, and risk assessment and epidemiology. Computer software was developed by the faculty for use in the course as were other resource materials. J. Cowan, S. Kurtz (Computer Science), and R. Thisted (Statistics) have worked on the courses.
5. The Sloan Foundation also funded the development of a quantitative methods course at SUNY Stony Brook taught by D. Ferguson- the Department of Technology and Society. The course builds mathematical models and uses these models to get approximate answers to questions which arise out of human need. Some examples of models are stock market simulation, drug testing, life insurance, and quality control in production. Computers are used in the course, and mathematical tools include probability and linear programming. Quantitative methods are portrayed as a "way of knowing" the world and mathematical techniques are developed in the context of real problems. The course uses three textbooks and notes, examples, and laboratory activities developed locally. The textbooks are Probability Examples by J. Truxal and N. Copp's {Vaccines: An Introduction to Risk both of which are in the New Liberal Arts Monograph Series, and How to Model It: Problem Solving for the Computer Age by A. Starfield, K. Smith, and A. Bleloch which is published by McGraw Hill.
6. Trinity College (Connecticut) developed a course "Essential Applications of Mathematics" aimed at enabling students to "be conversant and comfortable with the reasoning underlying such issues as environmental protection, the nuclear arms race, the spread of AIDS, and the budget deficit." The course focuses on five areas: numerical relations; proportions and percents; data analysis, probability and statistics; mathematical reasoning; and applications of algebra, geometry, and functions. Microcomputers are used. Laboratories are a part of the course. Materials have been developed locally. T.Craine and L. Deephouse have been involved in the course development which was linked to the adoption of a college mathematical proficiency requirement.
7. At Dartmouth College L. Snell and R. Prosser have developed with the participation of faculty at Grinnell, Middlebury, and Spelman College a course called CHANCE. The course develops concepts of probability and statistics only to the extent needed to understand the applications. An important part of the text material for the course is the journal *Chance* started by Springer-Verlag in 1988. Computer simulations and software packages are used. Units for study include maintaining quality of manufactured goods in the face of variation, and scoring streaks and records in sports. The New York Times is also a resource for the course, and writing is a means used in teaching the course.
8. Another course developed with the help of the Sloan Foundation is "Case Studies in Quantitative Reasoning: An Interdisciplinary Course" at Mount Holyoke College. H. Pollatsek and R. Schwartz have divided the course into the three units: I. Narrative and Numbers: Salem VillageWitchcraft;

II. Measurement and Prediction: SAT Scores and GPA;

III. Rates of Change: Modeling Population and Resources.

A three-hour weekly computer laboratory is an integral part of the course's structure. The emphasis is on reasoning and ways to construct and evaluate arguments. Besides study of probability and statistics, the course involves simple algebra, graph reading, linear and exponential models, rates of change,and simulation. Students are required to write three substantial papers in the course and do six laboratory reports besides homework exercises. Resource materials consist of a readings list rather than a specific textbook.

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Appendix D: Sample Materials

D-1 Beliefs About Mathematics and Problem Solving

Directions: For each item below circle the letter which corresponds to the measure of your agreement or disagreement with the statement.

1. Solving a mathematical problem usually relies on knowing some mathematical facts.
A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

2. Mathematics is mostly a set of rules for doing problems.
A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

3. Much mathematics is not really useful except for people who do very specialized work.
A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

4. Either people can do mathematics or people cannot do mathematics.
A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

5. There is only one correct answer to any mathematics problem.
A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

6. If a mathematical problem can be solved, it can usually be done in five minutes or less.
A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

7. There is only one way to do a mathematics problem correctly.
A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

8. To do mathematics is to calculate answers to problems.
A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

9. Learning mathematics involves mostly memorizing.
A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

10. Solving some mathematical problems involves knowing different strategies to try.
A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

D-2 Some Problems Related to Minimal Competency

1. Liam's bowling average is 134. Every Wednesday night at his league, Liam bowls five games. If his first three games are 130, 129, and 151, then what must Liam average in his last two games to keep his average at a 134 overall?

2. Sally knows that her Grandmother's five-digit zip code in south Florida begins with either "33" or `"34". How many zip codes has Sally narrowed it down to?

3. A 7-ounce can of pork and beans sells for $1.26. What is the highest price that a 20-ounce can may sell for and still be more economical than the smaller can?

4. Two neighbors, Wilma and Betty, each have a swimming pool. Both Wilma and Betty's pools hold 9000 gallons of water. If Wilma's garden hose fills at a rate of 650 gallons per hour while Betty's garden hose fills at a rate of 500 gallons per hour, then how much longer does it take Betty to fill her pool than Wilma?

5. The following appeared in a 1988 news magazine: How many alcoholic beverages did you drink in the past week? *(From a telephone poll of 664 people with sampling error +/- 4%).*

0	32%
1-7	49%
8-14	9%
15-19	1%
20-29	4%
30+	3%
no opinion	3%

6. The following paragraph is from USA TODAY, March 1992:

> *"I wonder whether we as a society fully understand and appreciate the subtext of the $650,000,000,000 we spend for health care? For example, how widely is it understood that total health care expenditures have risen 30\% faster than the GNP since 1970 while health care prices have increased 60% more rapidly than general price inflation? Is it fully appreciated that, despite this growth in the total amount of services, the even faster rise in their costs, and the $2,600 per capita we now are spending, access to basic health care services* ***remains*** *severely restricted for much of the population?"*

In a few sentences, describe what is meant by "...total health care expenditures have risen 30\% faster than the GNP since 1970..."

7. Connie has an unlimited supply of dimes and quarters. Her phone call at the public telephone is $.25 for the first minute and $.17 for each additional minute or portion thereof. She knows that her call will be 4 minutes and 30 seconds in length.

> *a. Determine the different combinations of coins that Connie can deposit to pay for her call. b. Explain which way (or ways) is most economical for her.*

8. To estimate the number of pigeons in a local park, city workers catch 82 and tag and release them. Later they catch 67 and find 20 of these tagged. How many pigeons should they expect to be in the park?

9. Jose has a refigerator with a freezer compartment which measures 26 inches by 15 inches by 15 inches. Freezer containers come in cartons of eight with each container a cube with 3.5 inches per edge. How many cartons must Jose buy to be sure he can fully utilize his freezer compartment?

10. On a loan of $1,000 Sophie is required to pay $45.91 for 24 monthly payments. Assuming interest is compounded monthly, what is the true annual interest rate of this loan?

11. Gloria noted that in June of 1989 her tuition to Super-U increased by 6% and in June of 1990 it increased by 3.5%. Over the two year span what was the percentage increase in tuition?

12. According to the Congressional Budget Office the actual expenditures for the federal government for the fiscal year 1992 were 1129 billion dollars. Anticipated expenditures for fiscal year 1993 are 1149 billion dollars, and the proposed federal budget for fiscal year 1994 is 1217 billion dollars. If a graph were drawn plotting expenditures versus time, would it have the appearance of a line segment connecting the point plotted for 1992 and the one plotted for 1994? Explain your answer in a complete sentence.

13. A laboratory technician has a large amount of a solution which is 40% alcohol and a large amount of a solution which is 60% alcohol. She would like to make 8 liters of a solution which is 55% alcohol. Determine the amount of 40% alcohol solution she will need and explain in a few sentences how you got your answer.

14. The Amtrack train is running 25 minutes behind schedule as it leaves Doverdale, traveling west to Topper Creek which is 78 miles away. If the train normally travels at 50 miles an hour, about how much faster would I have to travel to make up the lost time and arrive in Topper Creek on Time?

15. Harry raises tropical fish and noticed his guppy population increasing according to the following table:

Time in weeks	0	4	8
No. of guppies	3	33	363

If the guppies continue to increase at this rate, how many should he expect to have in twelve weeks? Explain your answer.

16. Dave works at a coffee shop. A customer asks for 1 1/2 lbs blend of 1/3 vanilla creme and 2/3 decaffienated Columbian. Dave grinds a blend of 2/3 vanilla creme with 1/3 decaffeinated Columbian. How can he give the customer what she asked for while wasting the least amount of

coffee? Explain your answer in a couple sentences.

15. Graduation pictures can be bought individually or in packages. Mary wants an 8 x 10, three 5 x 7's and 11 wallet size. Prices are listed below. What's the best deal for Mary? Explain your answer in a couple sentences.

Package A	Package B	Individual
1 - 8 x10	2 - 5 x 7	8 x 10 $18.00
10 wallets	12 wallets	5 x 7 $ 6.00
$25.00	$20.00	wallet $ 1.00

D-3 Project Ideas Project One: Chi-square Testing

You are to do this project working with your team of 4 students, but each student in the team is to do his/her own write-up of the project.
Your question to be considered is do male and female NIU students respond differently to the question of whether every American high school graduate should be able to complete a college degree. Determine to use the chi-square statistic to test the null hypothesis that there is no significant difference between male and female NIU students in response to Q: "Do you think every American high school graduate should be able to complete a college degree?"

Proceed as follows:

1. Determine the level of significance you (your team) will use for the test.
2. Determine how your team will obtain a good sample of 80-100 NIU students to be polled.
3. Conduct the poll of the sample asking the question Q and allowing the responses "Yes", "No", "No opinion". Tabulate responses.
4. Determine the expected distribution for responses to Q assuming the null hypothesis and the responses "Yes", "No", "No opinion".
5. Determine the number of degrees of freedom for the chi-square test of the null hypothesis and the value of the x2 statistic which would cause rejection of the null hypothesis.
6. Compute the x2 statistic for the data obtained and tell if the null hypothesis should be rejected.
7. Write up a short report on your study including a description of the points carried on in #1-6 and a brief paragraph discussing the findings of the study.

Project Two: Personal Business Applications (for collaborative study)

You are to do this project working with your team of 4 students, but each student in the team is to do his/her own write-up of it.

1. The Howies want to purchase a home for $98,000 and want a 30 year fixed rate mortgage for 90% of the purchase price. Three finance companies give quotes:

Center Bank	7% plus 1.75 points
First Federal	7.375% plus no points
Fleet Mortgage	7.25% plus one point

Which one should he choose and why?

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